Good afternoon Happy Thursday!

Agenda:

-Derivatives of e^x, sinx and cosx

-Product rule and quotient rule,
(last new topic for test)

-Quizzes returned next period

-We will later prove, the derivatives and rules just not today

$e \sim 2.7182\ldots$

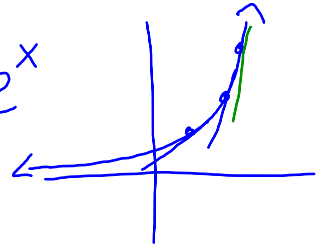
$$e = \lim_{x \to 0} (1+x)^{1/x} = P\left(1+\frac{r}{n}\right)^{n \cdot t}$$

$$Pe^{rt}$$

$2\pi r = C$

$\pi = \frac{C}{2r}$

$$\frac{d}{dx}\left(e^{x}\right) = e^{x}$$



$f(x) = e^{x} - x$

$f'(x) = \frac{d}{dx}(e^{x}) - \frac{d}{dx}(x)$

$\boxed{f'(x) = e^{x} - 1}$

$(\ln(2), 2)$

$y = e^{x}$

$y' = e^{x} = 2$ slope

$e^{\ln(2)} = \boxed{2}$

$\ln e^{x}$

$\boxed{x = \ln(2)}$

$y - 2 = e^{\ln(2)}(x - \ln(2))$

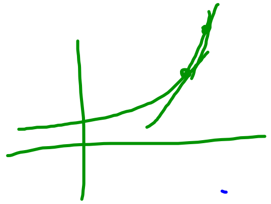
$\boxed{y - 2 = 2(x - \ln(2))}$

tangent line

$y - y_1 = m(x - x_1)$

$$e = \lim_{x \to 0}(1+x)^{1/x} \approx e$$

$$\frac{d}{dx}(e^x) = e^x$$



Ex)

$$f(x) = e^x - x$$

$$f'(x) = \frac{d}{dx}e^x - \frac{d}{dx}x$$

$$\boxed{e^x - 1}$$

Ex) $y = e^x = e^{\ln(2)} = \boxed{2}$ tangent line // to $y=2x$

$$y' = e^x = \boxed{2}$$

$$\boxed{x = \ln(2)}$$

$$e^{\ln(2)} = 2 = y$$

$$y - 2 = 2(x - \ln(2))$$

$$y - 2 = e^{\ln(2)}(x - \ln(2))$$

$f'(2)$

$$\frac{d}{dx}(\sin(x)) = \cos(x)$$

$$\frac{d}{dx}(\cos(x)) = -\sin(x)$$

$$g(x) = \cos(x) - \sin(x) + 3e^x$$

$$3\frac{d}{dx}e^x$$

$$g'(x) = \frac{d}{dx}\cos(x) - \frac{d}{dx}\sin(x) + \frac{d}{dx}3e^x$$

$$-\sin(x) - \cos(x) + 3e^x \qquad \frac{d}{dx}(3+e^x)$$

Product

$$\frac{d}{dx}\left(f(x)\cdot g(x)\right) = \left(f'(x)\,g(x)\right) + \left(g'(x)\,f(x)\right)$$

$$F'(x) = \left(\frac{d}{dx}f(x)\cdot g(x)\right) + \left(\frac{d}{dx}g(x)\cdot f(x)\right)$$

Ex 2) $F(x) = \underbrace{(x^2+3x)}_{f}\underbrace{(4x^3-6x)}_{g}$

$f'(x) = 2x+3 \qquad g'(x) = 12x^2-6$

$$F'(x) = \left[\underbrace{(2x+3)}_{f'}\underbrace{(4x^3-6x)}_{g}\right] + \left[\underbrace{(12x^2-6)}_{g'}\underbrace{(x^2+3x)}_{f}\right]$$

$$\left[8x^4-12x^2+12x^3-18x\right]+\left[12x^4+36x^3-6x^2-18x\right]$$

$$\boxed{20x^4+48x^3-18x^2-36x}$$

$f(x) = xe^x$

$x \cdot e^x$

$x \to h \qquad e^x \to g$

$h' = 1 \qquad g' = e^x$

$f'(x) = 1\cdot e^x + e^x \cdot x$

$= \boxed{e^x + xe^x}$ ✓ $\quad e^x(1+x)$

$$f(t) = \sqrt{t}\,(1-t)$$

$h \quad t^{\frac{1}{2}} \qquad\qquad g\,(1-t)$

$$h' = \frac{1}{2\sqrt{t}} \qquad\qquad g' = -1$$

$$f'(t) = \frac{1}{2\sqrt{t}} \cdot \frac{(1-t)}{1} + -1 \cdot \sqrt{t}$$

$$= \boxed{\frac{1}{2\sqrt{t}} - \frac{t^{5}}{2t^{1/2}} - \sqrt{t}} \qquad \frac{1}{2} \cdot t^{1-\frac{1}{2}}$$

$$\frac{1}{2} t^{\frac{1}{2}}$$

$$= \frac{1}{2\sqrt{t}} - \frac{1}{2}\sqrt{t} - \sqrt{t}$$

$$\checkmark \quad \boxed{\frac{1}{2\sqrt{t}} - \frac{3}{2}\sqrt{t}} \qquad \frac{1}{2}\sqrt{t^{9}}$$

Title: Oct 17-1:41 PM (6 of 17)

$$\sqrt{t}\,(1-t) \Rightarrow \sqrt{t} - t\sqrt{t}$$

$$t^{\frac{1}{2}} - t^{3/2} \qquad t^{1/2} - t^{1}(t^{1/2})$$

$$\frac{1}{2}t^{-\frac{1}{2}} - \frac{3}{2}t^{\frac{1}{2}}$$

$$\frac{1}{2t^{1/2}} - \frac{3}{2}t^{\frac{1}{2}} = \boxed{\frac{1}{2\sqrt{t}} - \frac{3}{2}\sqrt{t}}$$

$f(x) = \sqrt{x} \cdot g(x)$

$x^{\frac{1}{2}} = \frac{1}{2}x^{-\frac{1}{2}}$

$g(4) = 2$

$g'(4) = 3$

$f'(4) = ?$

$$\boxed{\frac{1}{2\sqrt{x}} \cdot g(x) + g'(x) \cdot \sqrt{x}}$$

$$\frac{1}{2\sqrt{4}} \cdot 2 + 3 \cdot \sqrt{4}$$

$$\frac{1}{4} \cdot 2 + 3 \cdot 2 = \frac{1}{2} + 6$$

$$\boxed{6.5 = \frac{13}{2}}$$

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{\left(g(x)\cdot\frac{d}{dx}f(x)\right) - \left(\left(f(x)\cdot\frac{d}{dx}g(x)\right)\right)}{(g(x))^2}$$

$$F'(x) = \frac{(g(x)\,f'(x)) - (f(x)\,g'(x))}{(g(x))^2}$$

$$y = \frac{x^2+x-2 \to f}{x^3+6 \to g}$$

$$y' = \frac{\left[\left(\overset{g}{x^3+6}\right)\overset{f'}{(2x+1)}\right] - \left(\left(\overset{f}{x^2+x-2}\right)\cdot \overset{g'}{3x^2}\right)}{(x^3+6)^2}$$

$$\frac{(2x^4+x^3+12x+6) - (3x^4+3x^3-6x^2)}{(x^3+6)(x^3+6)}$$

$$\boxed{\frac{-x^4-2x^3+6x^2+12x+6}{x^6+12x^3+36}}$$

$$\frac{d}{dx}\sin(x) = \cos(x)$$

$$\frac{d}{dx}\cos(x) = -\sin(x)$$

$$f(x) = \sin(x) + e^x$$

$$f'(x) = \cos(x) + e^x$$

$$g(x) = \cos(x) - \sin(x) + 3e^x$$

$$= \frac{d}{dx}\cos(x) - \frac{d}{dx}\sin x + \frac{d}{dx}3 \cdot e^x$$

$$g'(x) = -\sin(x) - \cos(x) + 3e^x$$

$$\frac{d}{dx}\left[f(x)\cdot g(x)\right]$$

$$=\left[\frac{d}{dx}(f(x))\cdot g(x)\right]+\left[\left(\frac{d}{dx}g(x)\right)\cdot f(x)\right]$$

$$F'(x)=(f'(x)\,g(x))+(g'(x)\,f(x))$$

$$F(x) = \underset{f}{(x^2+3x)}\underset{g}{(4x^3-6x)}$$

$$f'(x) = 2x+3 \qquad g'(x) = 12x^2-6$$

$$f'(x)\cdot g(x) + g'(x)\cdot f(x)$$

$$[(2x+3)(4x^3-6x)] + [(12x^2-6)(x^2+3x)]$$

$$8x^4-12x^2+12x^3-18x+(12x^4+36x^3-6x^2-18x)$$

$$\boxed{20x^4+48x^3-18x^2-36x}$$

$$F'(x) =$$

$f(x) = xe^x$

$h \qquad g$

$h'(x) = 1 \qquad g'(x) = e^x$

$h'(x) \cdot g(x) + g'(x) \cdot h(x)$

$1 \cdot e^x + e^x \cdot x$

$= \boxed{e^x + xe^x} = f'(x)$

$e^x(1+x)$

$$\frac{1}{2\sqrt{t}}\cdot(1-t) + -1(\sqrt{t})$$

$$\frac{1}{2\sqrt{t}} - \frac{t^{1}}{2t^{1/2}} + -\sqrt{t}$$

$$\frac{1}{2\sqrt{t}} - \frac{1}{2}\sqrt{t} - \sqrt{t} \qquad t^{1-\frac{1}{2}} = t^{\frac{1}{2}}$$

$$\boxed{\frac{1}{2\sqrt{t}} - \frac{3\sqrt{t}}{2}}\cdot\frac{\sqrt{t}}{\sqrt{t}}$$

$$\frac{1-3t}{2\sqrt{t}}$$

$f(x) = \sqrt{x} \cdot g(x)$ $\quad \rightarrow h$

$h' = \frac{1}{2\sqrt{x}}$

$g(4) = 2$

$g'(4) = 3$

$f'(4) = ?$

$h'(4)\,g(4) + g'(4)\,h(4)$

$\frac{1}{2\sqrt{4}} \quad 2 \quad 3 \cdot \sqrt{4}$

$\frac{d}{dx} x^{\frac{1}{2}} = \frac{1}{2} x^{-\frac{1}{2}}$

$= \frac{1}{2\sqrt{x}}$

$\frac{1}{4} \cdot 2 + 3 \cdot 2$

$\frac{1}{2} + 6 = 6.5$

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)\frac{d}{dx}f(x) - \left[f(x)\frac{d}{dx}g(x)\right]}{(g(x))^2}$$

$$F'(x) = \frac{g(x)f'(x) - (f(x)g'(x))}{(g(x))^2}$$

Ex

$f(x) = x^2 + x - 2$

$g(x) = x^3 + 6$

$$\frac{(x^3+6)(2x+1) - \left[(x^2+x-2)\cdot 3x^2\right]}{(x^3+6)^2}$$

$$\frac{\left[2x^4 + x^3 + 12x + 6\right] - (3x^4 + 3x^3 - 6x^2)}{(x^3+6)(x^3+6)}$$

$$\boxed{\frac{-x^4 - 2x^3 + 6x^2 + 12x + 6}{x^6 + 12x^3 + 36}}$$

HW:

1 to 12; 13, 17, 19