Good afternoon Happy Friday!

Agenda:
-Discuss Chain Rule

-Return Tests

-Prove derivative, of exponential.
-Give derivative of Logs
-Prove Sine, Cosine, Product and Quotient.
-Graph Sine and Cosine

Compositions

$f(g(x))$ $\qquad$ $g(f(x))$

$f \circ g$ $\qquad$ $g \circ f$

$f(x) = x^2$ $\qquad$ $g(x) = e^x$

$f(g(x)) = f(e^x) = \boxed{(e^x)^2} = e^{2x}$

$g(f(x)) = g(x^2)$

$e^{x^2} \neq (e^x)^x$

Chain Rule

$F = f(g(x)) \Rightarrow F' = \boxed{f'(g(x)) \cdot g'(x)}$

$y = f(u)$, $u = g(x)$ $\qquad$ $y' = \frac{dy}{du} \cdot \frac{du}{dx}$

$F = e^{2x} = (e^x)^2$

$u = e^x \rightarrow u' = e^x$

$y = u^2$

$y' = 2u$

$F'(u) = 2u \cdot u' = 2e^x \cdot e^x$

$= \boxed{2e^{2x}}$

$f(g(x))$

$f(x) = x^2$ $\qquad$ $g(x) = e^x$

$f'(x) = 2x$ $\qquad$ $g'(x) = e^x$

$2(e^x) \cdot e^x \Rightarrow \boxed{2e^{2x}}$

$f'(g(x)) \cdot g'(x)$

$\frac{d}{dx}\boxed{g(f(x))}$

$g(x) = e^x$

$f(x) = x^2$

$g'(x) = e^x$

$f'(x) = 2x$

$g'(f(x)) \cdot f'(x)$

$e^{x^2} \cdot 2x$

$\boxed{2xe^{x^2}}$

$f(u) = e^u$ $\qquad$ $u = x^2$

$u' = 2x$

$f'(u) \cdot u'$

$e^u \cdot u'$

$e^{x^2} \cdot 2x \rightarrow 2xe^{x^2}$

① $\sqrt{5x+4}$

$u = 5x+4 \quad = \quad \frac{du}{dx} \to u' = 5$

$f(u) = \sqrt{u} = u^{\frac{1}{2}}$

$\frac{dy}{du} = f'(u) = \frac{u^{-\frac{1}{2}}}{2} = \frac{1}{2u^{\frac{1}{2}}} \cdot 5$

$\frac{1}{2\sqrt{5x+4}} \cdot 5 = \boxed{\frac{5}{2\sqrt{5x+4}}}$

(2) $\sin(3x^2+x)$

$u = 3x^2+x$ $\qquad \frac{du}{dx} = 6x+1$

$f(u) = \sin(u)$ $\qquad f'(u)\cdot u'$

$\frac{dy}{du} = \cos(u)$ $\qquad \boxed{\cos(3x^2+x)\cdot(6x+1)}$

$f(g(x))$

(3) $y = \cos(e^x)$

$f(\cdot) = \cos$ $\qquad f'(g(x))\ -\sin(e^x)\cdot e^x$ g'

$g(x) = e^x$ $\qquad \boxed{-e^x\sin(e^x)}$

(4) $(44x-2x^3)^{1/3}$

$f(g) = (g(x))^{1/3} \rightarrow f'(g(x)) = \frac{1}{3}(g(x))^{-\frac{2}{3}}\cdot$

$g(x) = 44x-2x^3$ $\qquad g'(x) = 44-6x^2$

$\frac{1}{3(44x-2x^3)^{2/3}}\cdot 44-6x^2$

$\frac{44-6x^2}{3(44x-2x^3)^{2/3}} \rightarrow \frac{44-6x^2}{3\sqrt[3]{(44x-2x^3)^2}}$ ✓

① $(12x - 9x^8)^{46}$

$u = 12x - 9x^8$ $\quad y = u^{46} \to y' = 46u^{45}$

$u' = 12 - 72x^7$ $\quad 46(12x - 9x^8)^{45} \cdot (12 - 72x^7)$

③ $\cos^2(x)$

$(\cos(x))^2$ $\quad y = u^2 \quad y' = 2u$

$u = \cos(x)$ $\quad u' = -\sin(x)$

$-2\sin(x)\cos(x) = 2\cos(x) \cdot -\sin(x)$

④ $\sin(3x^2 - e^x)$ $\quad -2\cos x \sin x$

$u = 3x^2 - e^x \quad u' = 6x - e^x$

$y = \sin(u) \to y' = \cos(u)$

$\cos(3x^2 - e^x) \cdot (6x - e^x)$

$$\frac{d}{dx}e^x = e^x$$

$$e = \lim_{x\to 0}(1+x)^{1/x}$$

$$\lim_{h\to 0}\boxed{(1+h)^{1/h}} := e$$

$3x-3 \to 3(x-1)$

Euler's

$$\frac{d}{dx}e^x = \lim_{h\to 0}\frac{e^{x+h}-e^x}{h} = \frac{(e^x\cdot e^h)-e^x}{h}$$

$$e^x\lim_{h\to 0}\frac{(e^h-1)}{h} = \lim_{h\to 0}\frac{\left((1+h)^{1/h}\right)^h-1}{h}$$

$(3^2)^{3^-}$

$$e^x\lim_{h\to 0}\frac{(1+h)-1}{h} = e^x\lim_{h\to 0}\frac{h}{h}$$

e^x

$$\frac{d}{dx}(a^x) = \ln(a)a^x$$

implicit diff.

$$\frac{d}{dx}(\ln(x)) = \frac{1}{x}$$

$$\frac{d}{dx}\log_a(x) = \frac{1}{\ln(a)x}$$

$\frac{d}{dx}(f(x))$

$$\frac{4e^x(x^3-3x^2+2x-1)}{(x^3+2x+1)^2}$$

$\boxed{4e^x \cdot 3x^2}$

(d) $2x(\cos(x)) + -\sin(x)(x^2+3)$

$2x\cos x - x^2\sin x - 3\sin x$

(e) $e^x(\sin x + \cos x)$

$e^x \to \cancel{xe^{x-1}}$

$y - 2e = 4e(x-1)$

$2xe^x + 2e^x$

$\frac{1}{2\sqrt{x+6}}$

$3t^2 - 24t + 36$

$3(t^2 - 8t + 12)$

$(t-2)(t-6) = 0$

$t=2 \quad t=6$

$t^3 - 12t^2 + 36t = 0$

$f(g(x))$ $\qquad$ $g(f(x))$

$f \circ g$ $\qquad$ $g \circ f$

$f(x) = x^2 \quad g(x) = e^x$ $\qquad$ $\boxed{e^{x^2}}$

$f(g(x)) = (e^x)^2 = \boxed{e^{2x}}$ $\qquad$ $g(x^2) = e^{(x^2)} = e^{x^2}$

$$\frac{d}{dx}\left[f(g(x))\right] \Rightarrow \boxed{f'(g(x)) \cdot g'(x)}$$

$$f(u) = f'(u) \cdot u' = \frac{dy}{dx} \cdot \frac{du}{dx}$$

$u = g(x)$ $\qquad$ $\boxed{u = 2x}$

e^u

$F(x) = \boxed{e^{2x}}$ $\qquad$ $e^u u'$

$e^u \cdot u'$

$F'(x) = e^{2x} \cdot 2$ $\quad$ e^{2x}

$= 2\boxed{e^{2x}}$

$g(f(x)) = e^{x^2}$

$x^2 = u$

$y = e^u \to e^u \cdot u'$

$e^{x^2} \cdot 2x$

$\boxed{2xe^{x^2}}$

① $\sqrt{5x+4}$

$\boxed{U = 5x+4}$

$f(u) = \sqrt{u} = u^{\frac{1}{2}}$

$f'(u) \cdot u' = \frac{1}{2} u^{-\frac{1}{2}} \cdot u'$

$U = 5x+4$

$U' = 5$

$= \frac{1}{2u^{\frac{1}{2}}}$

$= \frac{1}{2\sqrt{5x+4}} \cdot 5$

$= \frac{5}{2\sqrt{5x+4}} = y'$

② $\sin(3x^2+x)$

$U = 3x^2+x \rightarrow U' = 6x+1$

$f(u) = \sin(u)$

$f'(u) \cdot u'$

$\cos(u) \cdot u' \rightarrow \cos(3x^2+x) \cdot (6x+1) = y'$

③ $y = \cos(e^x)$

$g(x) = e^x$

$f = \cos$

$f' = -\sin$

$g'(x) =$

$f(g(x))$

$\boxed{f'(g(x)) \cdot g'(x)}$

$-\sin(e^x) \cdot e^x$

④ $y = (44x - 2x^3)^{1/3}$

$g'(x) = -6x^2+44$

$g(x) = 44x - 2x^3$

$f' = \frac{1}{3(g(x))^{2/3}}$

$f = \sqrt[3]{u} = (x)^{1/3}$

$\frac{1}{3(44-2x^3)^{2/3}} \cdot (-6x^2+44) = \boxed{\frac{-6x^2+44}{3(44-2x^3)^{2/3}}}$

$3\sqrt[3]{(44-2x^3)^2}$

Title: Oct 25-11:42 AM (9 of 15)

① $(4x-18x^9)^{46}$

$u = 4x-18x^9$

$u' = 4-162x^8$

$u^{46} \to 46u^{45}\cdot u' = \boxed{46(4x-18x^9)^{45}\cdot(4-162x^8)}$

② $y = \sin(\underbrace{x+25x^3}_{u})$

$u = x+25x^3$

$u' = 75x^2+1$

$\sin(u) \to \cos(u)\cdot u' = \cos(x+25x^3) \times 75x^2+1$

$y' = \cos(x+25x^3)\cdot(75x^2+1)$

③ $\cos^2(x.) = (\cos(x))^2$

$u = \cos(x)$

$y = u^2 \Rightarrow$

$2u\cdot u'$

$\boxed{2\cos(x)\cdot -\sin(x)}$

$\boxed{-2\cos(x)\sin(x)}$

$$\frac{d}{dx} e^x = e^x \; *$$

$$\frac{d}{dx} a^x$$

$$e \to \lim_{x \to 0} (1+x)^{1/x}$$

$$\lim_{h \to 0} (1+h)^{1/h}$$

$$3x - 3 = 3(x-1)$$

$$\frac{d}{dx} e^x : \lim_{h \to 0} \frac{e^{x+h} - e^x}{h} = \frac{e^x \cdot e^h - e^x}{h}$$

$$e^x \lim_{h \to 0} \frac{(e^h - 1)}{h} \to \frac{\left((1+h)^{1/h}\right)^h - 1}{h}$$

$$e^x \lim_{h \to 0} \frac{((1+h)^{1/h})^h - 1}{h} = e^x \lim_{h \to 0} \frac{(\cancel{1}+h-\cancel{1})}{h}$$

$$(3^2)^3 = 3^6$$

$$e^x \lim_{h \to 0} \frac{h}{h} \Big\} 1$$

$$e^x$$

$$\frac{d}{dx} a^x = \ln(a) \cdot a^x$$

$$\frac{d}{dx} \ln(x) = \frac{1}{x}$$

implicit diff.

$$\frac{d}{dx} \log_a(x) = \boxed{\frac{1}{\ln(a)\,x}} = \frac{1}{\ln(a)} \cdot \frac{1}{x}$$

$$\frac{d}{dx}f(x)$$

$$D_x(f)$$

$$\frac{4e^x(x^3-3x^2+2x-1)}{(x^3+2x+1)^2}$$

$$-4e^x 3x^2$$

(d) $2x \cdot \cos x + (-x^2 \sin x - 3\sin x$

$2x\cos x - \sin x\,(x^2+3)$

(e) $e^x(\sin x + \cos x)$

$y - 2e = 4e(x-1)$

$$\frac{1}{2\sqrt{x+6}}$$

$\rightarrow 3(t^2 - 8t + 12)$

$(t-2)(t-6)$

$t = 2 \quad t = 6$