

Good morning Happy Tuesday!

Agenda:

-Warm Up

-Prove ax Sinx and Cosx

-Derivatives of other Transcendental functions

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$$= a^x \cdot \ln(a)$$
$$= \ln(a) \cdot a^x$$

$$y = 5^x \Rightarrow y' = \ln(5) \cdot 5^x$$

$$y = 3^{2x} = f'(u) \cdot u' = \ln(3) \cdot 3^u \cdot 2$$

$$u = 2x \quad f(u) = 3^u$$
$$\ln(3) (3^{2x}) \cdot 2$$
$$2 \cdot \ln(3) \cdot 3^{2x}$$
$$\ln(9) \cdot 3^{2x}$$

$$y = 4^{\sin(x)}$$

$$u = \sin x$$

$$f(u) = 4^u$$

$$f'(u) \cdot u'$$

$$\ln(4) \cdot 4^u \cdot \cos(x)$$

$$\ln(4) \cdot 4^{\sin x} \cdot \cos(x)$$

$$\ln(4) \cdot \cos(x) \cdot 4^{\sin x}$$

$$4 \cos(x) \cdot 4^{\sin x}$$

$$y = \underset{f}{2^{x^2}} \cdot \underset{g}{3^x}$$

$$g'(x) = \ln(3) \cdot 3^x$$

$$u = x^2 \quad u' = 2x$$

$$f(u) = 2^u$$

$$\ln(2) 2^u$$

$$\boxed{\ln(2) 2^{x^2} \cdot 2x} \quad f'$$

$$g f' + g' f$$

$$f' g + g' f$$

$$\left[2x \cdot 2^{x^2} \ln(2) \cdot 3^x \right] + \ln(3) \cdot 3^x \cdot 2^{x^2}$$

$$(\ln(2) 2x \cdot 2^{x^2} \cdot 3^x) + (\ln(3) 2^{x^2} 3^x)$$

$$\boxed{2^{x^2} 3^x (\ln(2) \cdot 2x + \ln(3))}$$

↓

$$\frac{d}{dx} \ln(x) = \frac{1}{x} \quad \frac{d}{dx} (\ln(1500x))$$

$$\frac{d}{dx} \ln(3x)$$

$u = 3x \rightarrow u' = 3$

$$f(u) = \ln(u) \rightarrow f'(u) = \frac{1}{u}$$

$$f'(x) = \frac{1}{3x} \cdot 3 = \frac{1}{x}$$

* $\frac{d}{dx} \ln(\cos x)$

$$f(u) = \ln(\) = \frac{1}{\cos x} = f'$$

$$g(x) = \cos(x) \quad g' = -\sin x$$

$$\frac{-\sin x}{\cos x} = \boxed{-\tan x}$$

$$\log_a(x) \rightarrow \frac{d}{dx} \log_a x = \frac{1}{\ln(a)x}$$

$$= \frac{1}{\ln(a)} \cdot \frac{1}{x}$$

Ex) $\frac{d}{dx} \log(x) = \boxed{\frac{1}{\ln(10)} \cdot \frac{1}{x}}$

$$\frac{d}{dx} \log_5(2x) =$$

$u = 2x \Rightarrow u' = 2$

$$f(u) = \log_5(u)$$

$$f'(u) = \frac{1}{\ln(5)u}$$

$\frac{1}{2x \ln(5)} \cdot 2$
 $\frac{1}{x \ln(5)}$

$$\frac{d}{dx} \sin x = \cos x$$

$$\lim_{h \rightarrow 0} \frac{\sin(x+h) - \sin(x)}{h}$$

$$\lim_{h \rightarrow 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$$

$$\lim_{h \rightarrow 0} \frac{\cos x \sin h (-\sin x + \sin x \cos h)}{h}$$

$$\lim_{h \rightarrow 0} \frac{\cos x \sin h}{h} + \lim_{h \rightarrow 0} \frac{-\sin x (1 - \cos h)}{h}$$

$$\cos x \lim_{h \rightarrow 0} \frac{\sin h}{h} + -\sin x \lim_{h \rightarrow 0} \frac{1 - \cos h}{h}$$

$$\boxed{\cos x}$$

In words explain

1)

a) What a derivative is/represents...

^{at}
~~Slope~~ of a point in a function
instantaneous rate.

b) What the power rule is...

Way to derive a power function
Find derivatives using polynomials

c) What a tangent line represents

Slope at point

• Line intersecting at one point, instant rate

2) Answer a, b, and c from part 1 but in 8 words or less.

$$e^x$$

$$a^x \rightarrow \frac{d}{dx} a^x = \ln(a) \cdot a^x$$

$$\frac{d}{dx} \left(e^{\ln(a^x)} \right) = \frac{d}{dx} e^{x \ln(a)}$$

$$u = x \ln(a) \quad f'(u) \cdot u'$$
$$f(u) = e^u \rightarrow f'(u) = e^u \quad u' = \ln(a)$$

$$e^u \cdot \ln(a)$$
$$e^{x \ln(a)} \cdot \ln(a)$$
$$e^{\ln(a)^x} \cdot \ln(a)$$

$$a^x \cdot \ln(a) = \ln(a) \cdot a^x$$

$$\text{Ex 1} \quad 3^x \Rightarrow \frac{d}{dx} 3^x$$

$$\ln(3) \cdot 3^x$$

$$\text{Ex 2} \quad 5^{x^2} \rightarrow \frac{d}{dx} 5^{x^2}$$

$$u = x^2$$

$$f(u) = 5^u$$

$$f'(u) = \ln(5) 5^u$$

$$\ln(5) 5^u \cdot 2x$$

$$\ln(5) \cdot 2x \cdot 5^{x^2}$$

$$x \ln(25) 5^{x^2}$$

$$(2 \ln(5) x) 5^{x^2}$$

$$\text{Ex 3} \quad 4^{\sin(x)}$$

$$u = \sin(x)$$

$$f(u) = 4^u$$

$$f'(u) = \ln(4) \cdot 4^u$$

$$\ln(4) \cdot 4^{\sin(x)} \cdot \cos(x) = \ln(4) \cos(x) 4^{\sin(x)}$$

Ex 5

$$y = 2^{4x} \cdot 3^{2x}$$

$f \swarrow \quad \searrow g$

$f'g + g'f$

$$f' = \ln(2) \cdot 2^{4x} \cdot 4 \quad (4)$$

$$g' = \ln(3) \cdot 3^{2x} \cdot 2 \quad (2)$$

$$\ln(2) \cdot 4 \cdot 2^{4x} \cdot 3^{2x} + \ln(3) \cdot 2 \cdot 3^{2x} \cdot 2^{4x}$$

$$[2^{4x} \cdot 3^{2x} \cdot 2] (2 \ln(2) + \ln(3)) \checkmark$$

$$2^{4x} \cdot 3^{2x} \left[\begin{array}{l} 4 \ln(2) + 2 \ln(3) \\ \ln(16) + \ln(9) \\ \ln(144) \end{array} \right]$$

$$2^{4x} \cdot 3^{2x} \ln(144)$$

$$\frac{d}{dx} \ln(x) = \frac{1}{x}$$

$$\frac{d}{dx} \ln(5x)$$

$$u' = 5$$

$$u = 5x$$

$$f(u) = \ln(u)$$

$$f'(u) = \frac{1}{u}$$

$$\frac{1}{5x} \cdot 5 = \frac{5}{5x} = \frac{1}{x}$$

$$\ln(100x)$$

$$u = 100x$$

$$f(u) = \ln(u)$$

$$f' = \frac{1}{u} \cdot u' = \frac{1}{100x} \cdot 100 = \frac{1}{x}$$

$$\text{Ex) } \ln(\cos(x))$$

$$u = \cos(x)$$

$$u' = -\sin(x)$$

$$f(u) = \ln(u)$$

$$f'(u) = \frac{1}{u}$$

$$f'(x) = \frac{1}{\cos(x)} \cdot -\sin(x)$$

$$\frac{-\sin(x)}{\cos(x)} = -\tan(x)$$

Ex 11

$$\frac{d}{dx} \log_7(x^2)$$

$$u = x^2$$

$$u' = 2x$$

$$f(u) = \log_7(u)$$

$$f'(u) = \frac{1}{\ln(7) \cdot u}$$

$$f'(x) = \frac{2x}{\ln(7) \cdot x^2} = \frac{2}{\ln(7) \cdot x}$$

$$\frac{d}{dx} \sin x = \cos x$$

$$\lim_{h \rightarrow 0} \frac{\sin(x+h) - \sin x}{h}$$

$$\lim_{h \rightarrow 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$$

$$\lim_{h \rightarrow 0} \frac{\cos x \sin h - \sin x + \sin x \cos h}{h}$$

$$\lim_{h \rightarrow 0} \frac{\cos x \sin h}{h} + \lim_{h \rightarrow 0} \frac{-\sin x + \sin x \cos h}{h}$$

$$\lim_{h \rightarrow 0} \frac{\cancel{\cos x} \sin h}{h} + \lim_{h \rightarrow 0} \frac{-\sin x (1 - \cos h)}{h}$$

$$\cos x (1) + \sin x (0)$$

$$\cos x$$

↓
0

41. $f(t) = t^2 \sin t$

43. $f(t) = \frac{\cos t}{t}$

45. $f(x) = -e^x + \tan x$

47. $g(t) = \sqrt[4]{t} + 6 \csc t$

49. $y = \frac{3(1 - \sin x)}{2 \cos x}$

51. $y = -\csc x - \sin x$

53. $f(x) = x^2 \tan x$

55. $y = 2x \sin x + x^2 e^x$

57. $y = \frac{e^x}{4\sqrt{x}}$

42. $f(\theta) = (\theta + 1)\cos \theta$

44. $f(x) = \frac{\sin x}{x^3}$

46. $y = e^x - \cot x$

48. $h(x) = \frac{1}{x} - 12 \sec x$

50. $y = \frac{\sec x}{x}$

52. $y = x \sin x + \cos x$

54. $f(x) = \sin x \cos x$

56. $h(x) = 2e^x \cos x$

58. $y = \frac{2e^x}{x^2 + 1}$

