Good morning Happy Tuesday!

Agenda:
-Warm UP

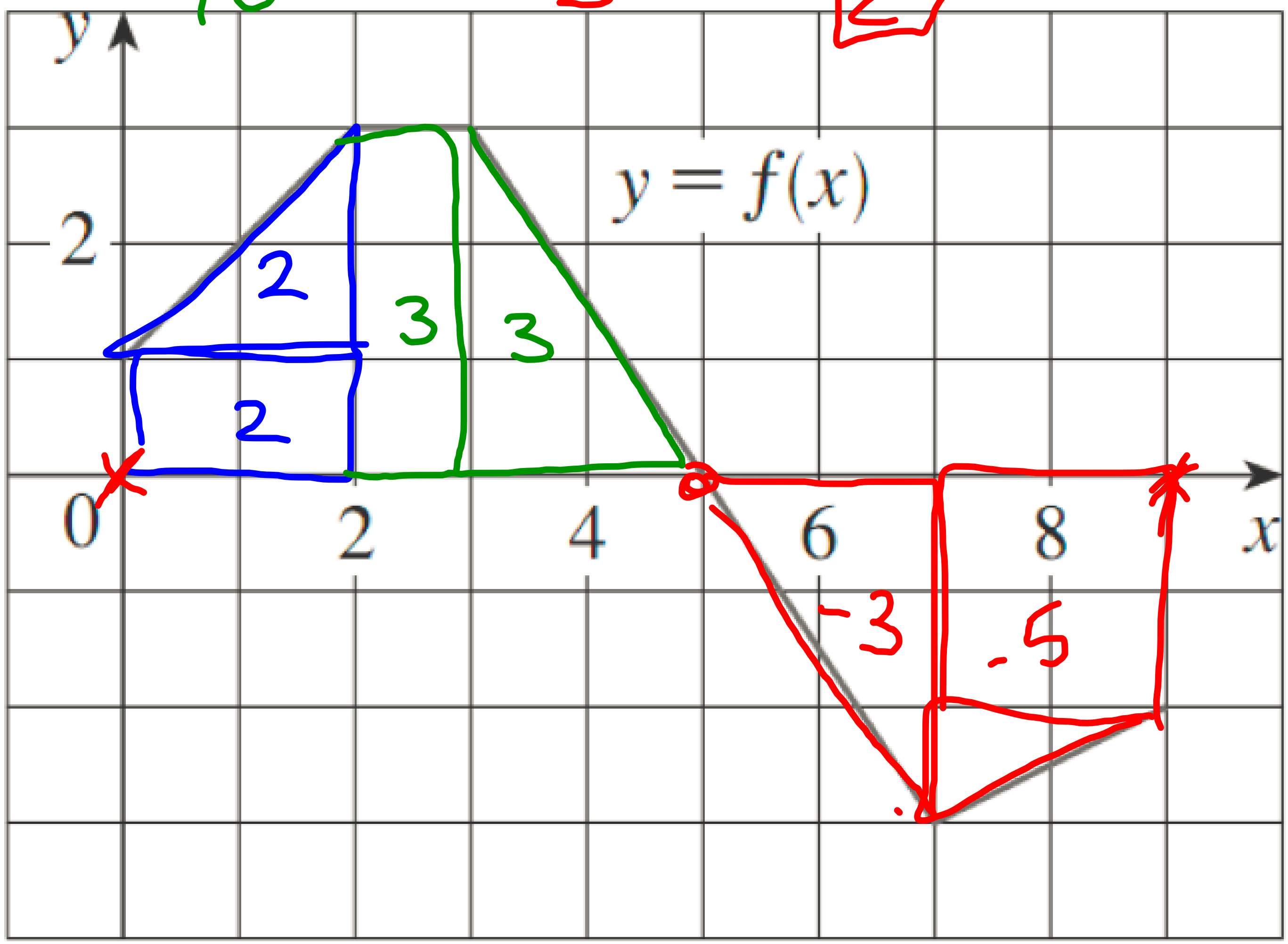
-Definite Integrals and Antiderivative notes (you should have printed it out or you can find on the web site)

-Quizzes will be returned next class, when we learn about Fundamental Theorem of Calculus.

Warm UP

The graph of f is shown below. Evaluate each integral by interpreting it in terms of areas.

a) $\int_0^2 f(x)\,dx$ 4 b) $\int_0^5 f(x)\,dx$ 10 c) $\int_5^7 f(x)\,dx$ -3 d) $\int_0^9 f(x)\,dx$ $\boxed{2}$



Rules for Definite Integrals			
#	**Rule**	**Notation**	**Statement**
1.	Order of Integration	$\int_a^b f(x)dx = -\int_b^a f(x)dx$	If you reverse the order of integration, you get the opposite answer.
2.	Zero	$\int_a^a f(x)dx = 0$	This should make sense if you think about the area of a rectangle with no width.
3.	Constant Multiple	$\int_a^b k \cdot f(x)dx = k \cdot \int_a^b f(x)dx$	Taking the constant out of the integral many times makes it simpler to integrate.
4.	Sum and Difference	$\int_a^b [f(x) \pm g(x)]dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$	This allows you to integrate functions that are added or subtracted separately.
5.	Additivity	$\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$	Pay close attention to the limits of integration ... this comes in handy when dealing with total area or other functions where we need to break them into smaller parts.

$$\int_6^{11} f(x)\,dx = -7 \text{ and } \int_6^{11} g(x)\,dx = 24.$$

a) $\int_{11}^{6} 9f(x)\,dx$

$$-\int_6^{11} 9f(x)\,dx$$

$$\underbrace{-9\int_6^{11} f(x)\,dx}$$

$$-9(-7) = 63$$

b) $\int_6^{11} 6g(x) - 10f(x)\,dx$

$$\int_6^{11} 6g(x)\,dx - \int_6^{11} 10f(x)\,dx$$

$$6\underbrace{\int_6^{11} g(x)\,dx} - 10\underbrace{\int_6^{11} f(x)\,dx}$$

$$6\,\boxed{24} - 10(-7)$$

$$144 - -70$$

$$214$$

Determine the value of $\int_2^9 f(x)\,dx$ given that $\int_5^2 f(x)\,dx = 3$ and $\int_5^9 f(x)\,dx = 8$.

$$-\int_2^5 f(x)\,dx + \int_5^9 f(x)\,dx$$

$$\int_2^9 f(x)\,dx = -3 + 8 = \boxed{5}$$

e the value of $\int_{-4}^{20} f(x)\,dx$ given that $\int_{-4}^{0} f(x)\,dx = -2$, $\int_{31}^{0} f(x)\,dx = 19$ and $\int_{20}^{31} f(x)\,dx = -21$.

$$\int_{-4}^{0} f(x)\,dx + -\int_{0}^{31} f(x)\,dx + -\int_{31}^{20} f(x)\,dx$$

$$\int_{-4}^{20} f(x)\,dx = -2 + -19 + 21 = \boxed{0}$$

The Average Value of a Function

If f is integrable on [a, b], its average value on [a, b] is given by

$$Average\ Value = \frac{1}{b-a}\int_a^b f(x)dx \text{ or } Average\ Value = \frac{\int_a^b f(x)dx}{b-a}$$

The average value of a function is just the integral over the interval.

To get a more geometric idea of what the average value is, complete the following example.

b-a : ? Length Of interval

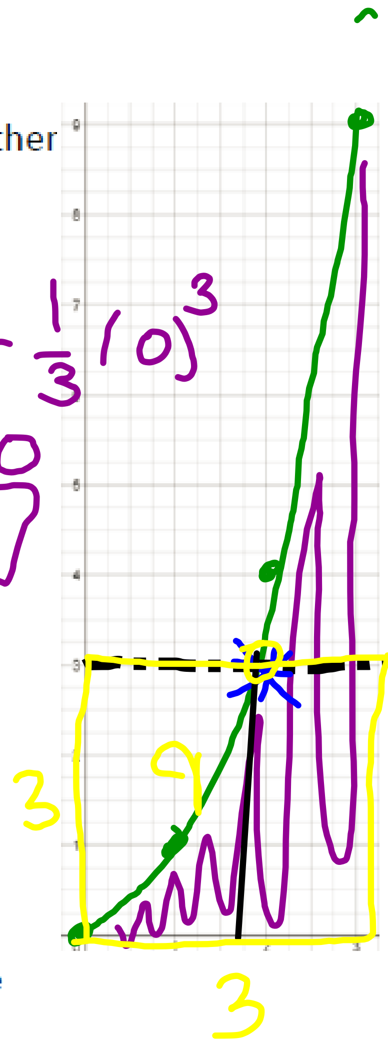
$\int_a^b f(x)dx$: $\dfrac{\text{Total Area}}{\text{Length of int.}}$ $\dfrac{\sum \text{Area rectangle}}{}$ = # of rect.

a) Graph the function $y = x^2$ on [0, 3] on the grid to the right.

b) Set up a definite integral to find the average value of y on [0, 3], ther calculator to evaluate the definite integral.

$$\int_0^3 x^2\,dx \rightarrow \frac{1}{3}x^3 \Big|_0^3 \Rightarrow \frac{1}{3}(3)^3 - \frac{1}{3}(0)^3$$

$$9 - 0 = \boxed{9}$$



Avg value

$$\frac{1}{3-0}\int_0^3 x^2\,dx = \frac{1}{3}(9) = \boxed{3}$$

c) Graph this value as a function on the grid to the right. Does this function ever actually equal this value? If so, at what point(s) in the interval does the function assume its average value?

$$x^2 = 3 \qquad x = \pm\sqrt{3} = \sqrt{3}$$

d) What do you suppose is the relationship between the area the x-axis and the curve $y = x^2$ on [0, 3] and the area of the rectangle formed using the average value as the height and the interval [0, 3] as the width?

They are the same!

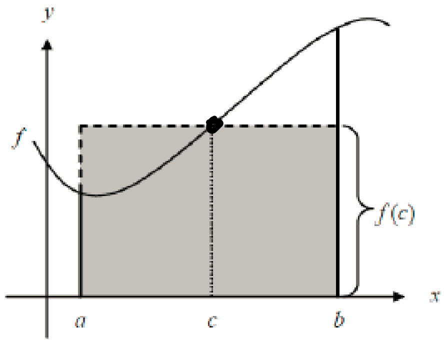
The Mean Value Theorem for Definite Integrals

The Mean Value Theorem for Integrals basically says that if you are finding the area under a curve between x = a and x = b, then there is some number c between a and b whose function value you can use to form a rectangle that has an area equal to the area under the curve.

Example 8

a) What is an expression that could be used to determine the area under the curve from a to b?

$$\int_a^b f(x)\,dx$$



b) What is the area of the shaded rectangle?

$$f(c)(b-a)$$

This value of $f(c)$ is just the average value of f on the interval [a, b].

$$f(c)(b-a) = \int_a^b f(x)\,dx$$

$$f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$$c^2 = 3$$

$$c = \sqrt{3}$$

Find Avg. Value

$y = -3x^2 - 1 \quad [0, 2]$

$\frac{1}{2-0}\int_0^2 -3x^2 - 1\,dx \Rightarrow \frac{1}{2}\left[-x^3 - x\right]\Big|_0^2$

$\frac{1}{2}\left[\left(-(2)^3 - 2\right) - \left(-0^3 - 0\right)\right]$

$\frac{1}{2}\left[(-8-2) - 0\right] = \frac{1}{2}\left[-10\right] = -5$

Find c, that works with MVT Int.

$f(c) = -5$

$-3(c)^2 - 1 = -5 \qquad c^2 = \frac{-4}{-3} = \frac{4}{3}$

$\quad\quad\quad +1 \quad +1$

$c^2 = \frac{4}{3} \qquad \boxed{c = \sqrt{\frac{4}{3}}}$

$\boxed{c = \frac{2\sqrt{3}}{3}}$

Kahoot: 826932

Ex 4

a) Find Avg value

$y = -3x^2 - 1 \quad [0, 2]$