Good afternoon Happy Monday!

Agenda:
-Warm UP

-Fundamental Theorem of Calculus Part 2

- Review problems/Answer questions

Evaluate the integrals below

1) $\int_2^3 x^4 + 4x + 6 \, dx$ $\quad \frac{1}{5}x^5 + 2x^2 + 6x \Big|_2^3$

2) $\int_\pi^{2\pi} (1 - \sin x) dx$

$\left[\frac{3^5}{5} + 2(3)^2 + (6(3))\right) = 58.2$

$- \left[\frac{2^5}{5} + 2(2)^2 + (6(2))\right]$

$\frac{1}{2}\int \frac{1}{x} dx$

3) $\int \frac{3}{\sqrt[3]{x^2}} + 2^x \, dx$

$\int 3x^{-\frac{2}{3}} + 2^x dx$

$9x^{\frac{1}{3}} + \frac{2^x}{\ln(2)} + C$

4) $\int \csc^2 x + \frac{1}{2x} dx$

$-\cot x + \frac{1}{2}\ln|x| + C$

Given $\int_3^8 g(x)\,dx = 10$ & $\int_3^1 g(x)\,dx = -4$

$-\int_1^3 g(x)dx$

Find

a) $\int_1^3 g(x)\,dx = 4$

b) $\int_1^8 \frac{1}{2} g(x)\,dx = 7$

c) $\int_3^3 g(x)\,dx = 0$

We are going to determine how to take a derivative of a function that is defined as an integral and discuss what it means to define a function as an integral.

Once we can do both of these things, we can answer all the same types of questions about increasing, decreasing, concave up, concave down, and inflection points that we did earlier in the year

Using the
Fundamental Theorem of Calculus part 1

$\int_a^b f(x)dx = F(b) - F(a)$

2 examples

EX

$$\frac{d}{dx}\left[\int_2^x g'(t)\,dt\right] = g(t)\Big|_2^x$$

$$\frac{d}{dx}\left(g(x) - g(2)\right)$$

$$g'(x) - 0 = \boxed{g'(x)}$$

$$\frac{d}{dx}\int_0^x \sin\theta \, d\theta$$

$$\frac{d}{dx}\left[-\cos\theta \Big|_0^x\right]$$

$$-(-\sin x)$$

$$\frac{d}{dx}\left[-\cos x - 1\right]$$

$$\sin x + 0$$

$$\boxed{\sin x}$$

The Second Fundamental Theorem of Calculus

If f is continuous on an open interval containing a, then, for every x in the interval,

$$\frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = f(x)$$

Ex)

$$\frac{d}{dx}\int_3^x 5t^3 - 4t + 1\,dt$$

$$\boxed{5x^3 - 4x + 1}$$

$$\frac{d}{dx}\left[\frac{5}{4}t^4 - 2t^2 + t \Big|_3^x\right]$$

$$\frac{d}{dx}\left[\left[\frac{5}{4}(x)^4 - 2(x)^2 + x\right] - \left(\frac{5}{4}(3)^4 - 2(3)^2 + 3\right)\right]$$

$$5x^3 - 4x + 1$$

$$\frac{d}{dx}\int_{g(x)}^{f(x)} h'(t)\,dt = \frac{d}{dx}\left[h(t)\Big|_{g(x)}^{f(x)}\right]$$

$$\frac{d}{dx}\left[h(f(x)) - h(g(x))\right]$$

$$\boxed{h'(f(x))\cdot f'(x) - h'(g(x))\cdot g'(x)}$$

3 Examples

The Second Fundamental Theorem of Calculus Extended

If f is continuous on an open interval containing a, then, for every x in the interval,

$$\frac{d}{dx}\left[\int_{h(x)}^{g(x)} f(t)\,dt\right] = f(g(x))g'(x) - f(h(x))h'(x)$$

2 Examples

$$\frac{d}{dx}\int_{x}^{x^2} t^{\frac{1}{2}} - t^2 \, dt$$

$$\frac{d}{dx}\left[\frac{2}{3}t^{\frac{3}{2}} - \frac{1}{3}t^3 \Big|_{x}^{x^2}\right]$$

$$\frac{d}{dx}\left[\frac{2}{3}(x^2)^{3/2} - \frac{1}{3}(x^2)^3 - \left(\frac{2}{3}x^{\frac{3}{2}} - \frac{1}{3}x^3\right)\right]$$

$$\frac{d}{dx}\left[\frac{2}{3}x^3 - \frac{1}{3}x^6 - \frac{2}{3}x^{\frac{3}{2}} + \frac{1}{3}x^3\right]$$

$$2x^2 - 2x^5 - x^{\frac{1}{2}} + x^2$$

$$\boxed{-2x^5 + 3x^2 - x^{\frac{1}{2}}}$$

Ex)

$$\frac{d}{dx}\int_{2x}^{x^3} (t^2 - 3t^5)\,dt$$

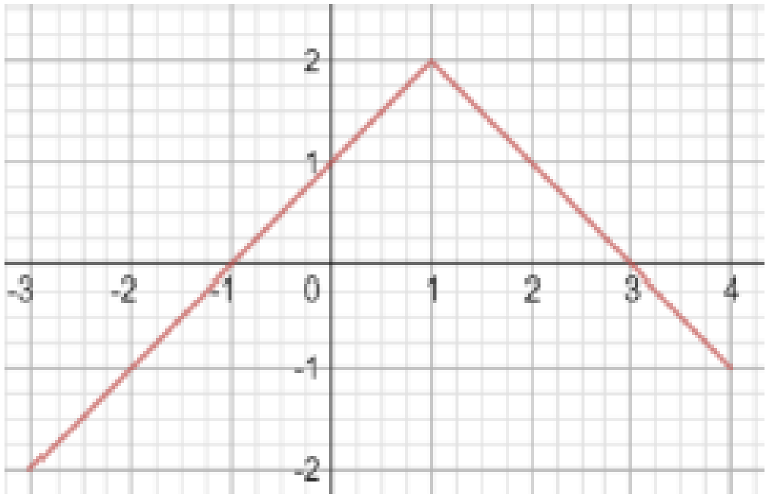
$$[(x^3)^2 - 3(x^3)^5]3x^2 - [(2x)^2 - 3(2x)^5]\cdot 2$$

$$[x^6 - 3x^{15}]\cdot 3x^2 - [4x^2 - 96x^5]\cdot 2$$

$$3x^8 - 9x^{17} - 8x^2 + 192x^5$$

Let $g(x) = \int_{-2}^{x} w(t)\,dt$ where the graph of $w(t)$ is given below.

a) Find $g(0)$.



b) Find $g(2)$.

c) Find $g(-3)$.

Given $f(x) = 1/3x^3+x^2$

a) Find the x-coordinates of the critical points.

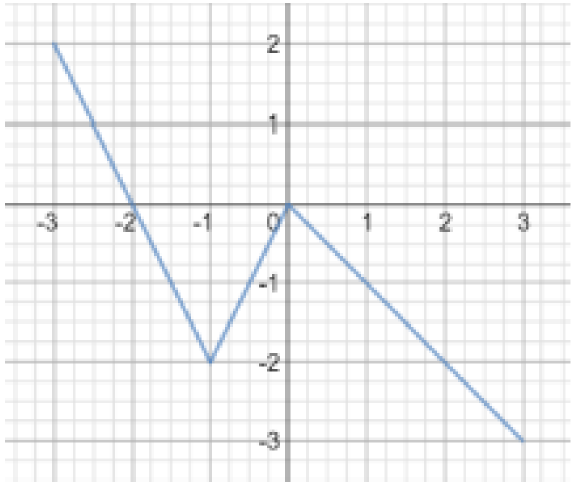
b) Find the x-coordinates of any minima or maxima.

c) Find the open intervals where the function is either increasing or decreasing. Justify.

d) Find any inflection points. Justify.

e) Find the open intervals where the function either concave or concave down. Justify. `

Suppose the function below is the graph of $f(t)$ and $g(x) = \int_{-1}^{x} f(t)\, dt$.



a) Complete the table.

x	-3	-2	-1	0	1	2	3
g(x)							

b) What are the intervals on which g is increasing or decreasing? Justify each response.

c) What are the intervals on which g is concave up or concave down? Justify each response.

d) For what value of x does g have a relative minimum or maximum? Justify your response.

e) For what value of x does g have an inflection point? Justify your response.

Review: **Find the average value of the function on the specified interval without a calculator.**

A) $g(x)=9-3x^2$ on the interval $[0, 4]$

B) $h(x)=\csc(x)\cot(x)$ on the interval $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$

C)

Including start-up costs, it costs a printer $50 to print 24 copies of a newsletter, after which the marginal cost (in ars per copy) at x copies is given by $C'(x)=\frac{2}{\sqrt{x}}$. Find the total cost of printing 2500 newsletters.

D) If you know $\int_{-7}^{9} f'(x)\,dx=15$, and you know $f(-7)=4$, what does $f(9)=?$

A 2.0) Find the value *c* guaranteed by the MVT.