Good morning Happy Friday!

Agenda:
-Warm Up

-More advanced practice

-Integral of Trig and what results will give us inverse trig

-Rest of Semester Schedule

Find the antiderivative of

1) $\int (5x+2)^7 \, dx$

2) $(\sin(\ln(x))/x = \frac{\sin(\ln(x))}{x}$

$u = 5x+2$

$\frac{du}{dx} = 5$

$du = 5dx$

$\frac{1}{5} du = dx$

$\int u^7 \cdot \frac{1}{5} du$

$\frac{1}{5} \int u^7 du$

$\frac{1}{5} \cdot \frac{1}{8} u^8 + C$

3) $(2x-3)/(x^2-3x+1)^{1/2}$ $\boxed{\int \frac{2x-3}{(x^2-3x+1)^{\frac{1}{2}}} dx}$

$u = (x^2-3x+1)$

$du = \boxed{2x-3 \, dx}$

$\int \frac{1}{u^{\frac{1}{2}}} du = 2u^{\frac{1}{2}} + C = \boxed{2\sqrt{x^2-3x+1} + C}$

$$\int_0^{2} \sqrt{4x+1}\, dx$$

$u = 4x+1$

$du = 4\, dx$

$\frac{1}{4} du = dx$

$u = 4(2)+1 = 9$

$u = 4(0)+1 = 1$

$$\frac{1}{4}\int_1^{9} u^{\frac{1}{2}}\, du = \frac{1}{4}\cdot\frac{2}{3}\, u^{\frac{3}{2}} \Big|_1^9$$

$$\frac{1}{6}(9)^{\frac{3}{2}} - \frac{1}{6}(1)^{3/2}$$

$$\frac{27}{6} - \frac{1}{6} = \frac{26}{6} = \frac{13}{3}$$

$$\frac{1}{6}(4x+1)^{3/2}\Big|_0^2$$

$u = 2(0)^2+3$

$u = 2(-3)^2+3$

$= 21$

$$-\int_{-3}^{0} \frac{8x}{(2x^2+3)^2}\, dx$$

$u = 2x^2+3$

$2(du = 4x\, dx)$

$$-2\int_{21}^{3} \frac{1}{u^2}\, du = 2\int_3^{21} u^{-2}\, du \quad \frac{u^{-1}}{-1}$$

$$2 \cdot \frac{-1}{u}\Big|_3^{21} \Rightarrow -\frac{2}{21} - -\frac{2}{3}\cdot\frac{7}{7}$$

$$-\frac{2}{21} + \frac{14}{21} = \frac{12}{21}$$

$$\boxed{\int \tan\theta \, d\theta}$$

$$\int \frac{\sin\theta}{\cos\theta} d\theta$$

$u = \cos\theta$

$du = -\sin\theta \, d\theta$

$$-\int \frac{1}{u} du = -\ln|u| + C$$

$$= \boxed{-\ln|\cos\theta| + C}$$

$$= \ln|(\cos\theta)^{-1}| + C$$

$$= \boxed{\ln|\sec\theta| + C}$$

$$\boxed{\int \cot\theta \, d\theta} = \boxed{\begin{array}{l} -\ln|\csc\theta| + C \\ \ln|\sin\theta| + C \end{array}}$$

$$\boxed{\int \sec\theta \, d\theta} \cdot \left(\frac{\sec\theta + \tan\theta}{\sec\theta + \tan\theta}\right)$$

$$\int \frac{\sec^2\theta + \sec\theta\tan\theta}{\sec\theta + \tan\theta} d\theta$$

$U = \sec\theta + \tan\theta$

$du = \sec\theta\tan\theta + \sec^2\theta \, d\theta$

$$\int \frac{1}{u} du = \ln|u| + C$$

$$\boxed{\ln|\sec\theta + \tan\theta| + C}$$

Ex | $\int \sec(3\theta) - 1 \, d\theta$

$\int \sec(3\theta)\,d\theta - \int 1 \, d\theta$ $\quad u = 3\theta$, $du = 3d\theta$

$\frac{1}{3}\int \sec(u)\,du - \theta + C$

$\frac{1}{3}\ln|\sec(u) + \tan(u)| - \theta + C$

$\frac{1}{3}\ln|\sec(3\theta) + \tan(3\theta)| - \theta + C$

Ex | $\int \cot\left(\frac{\theta}{3}\right) d\theta$ $\quad u = \frac{1}{3}\theta$, $3\left(du = \frac{1}{3}d\theta\right)$

$3\int \cot(u)\,du$

$3\ln|\sin(u)| + C$ $\quad -3\ln|\csc(u)| + C$

$\boxed{3\ln\left|\sin\left(\frac{1}{3}\theta\right)\right| + C}$

$\int e^{\csc\theta} d\theta$ $\quad U = \csc\theta$, $du = -\csc\theta\cot\theta \, d\theta$

$\int \frac{-\csc\theta\cot\theta}{e^{\csc\theta}} d\theta$ $\quad U = \csc\theta$, $du = -\csc\theta\cot\theta \, d\theta$

$\int \frac{1}{e^u} du$

Title: Mar 6-1:59 PM (5 of 9)

$$\int_{-3}^{0} -\frac{8x}{(2x^2+3)^2}\,dx$$

$u = 2x^2+3$

$2(du) = (4x\,dx)2$

$2\,du = 8x\,dx$

$2(0)^2+3 = 3$

$2(-3)^2+3 = 21$

$$-\int \frac{1}{u^2}\cdot 2\,du = -2\int_{21}^{3} \frac{1}{u^2}\,du$$

$$2\int_{3}^{21} \frac{1}{u^2}\,du \qquad -\frac{2}{u}\Big|_{3}^{21} \qquad -\frac{2}{21} - -\frac{2}{3}\cdot\frac{7}{7}$$

$$u^{-2} \rightarrow \frac{u^{-1}}{-1} \qquad -\frac{2}{21} + \frac{14}{21} = \frac{12}{21}$$

Title: Mar 6-11:59 AM (6 of 9)

Trig

$\int \frac{1}{\cos\theta} \cdot \sin\theta\, d\theta$

$$\int \tan\theta\, d\theta = \int \boxed{\frac{\sin\theta}{\cos\theta}\, d\theta}$$

$$\left.\begin{aligned} u &= \cos\theta \\ du &= -\sin\theta\, d\theta \end{aligned}\right\} -\int \frac{1}{u}\, du = -\ln|u| + c$$

$(\cos\theta)^{-1}$

$$\boxed{-\ln|\cos\theta| + c}$$

$$\ln\left|\frac{1}{\cos\theta}\right| + c =$$

$$\boxed{\ln|\sec\theta| + c}$$

$$m \log_b(x) = \log_b(x^m)$$

$$\int \cot\theta\, d\theta = \ln|\sin\theta| + c$$

$\frac{1}{\frac{1}{\sin}}$

$$-\ln|\csc\theta| + c$$

$$\int \sec\theta\, d\theta \frac{(\sec\theta + \tan\theta)}{(\sec\theta + \tan\theta)}$$

$$\int \frac{\sec^2\theta + \sec\theta\tan\theta}{(\sec\theta + \tan\theta)}\, d\theta$$

$u = \sec\theta + \tan\theta$

$du \boxed{= \sec\theta\tan\theta + \sec^2\theta\, d\theta}$

$$\int \frac{1}{u}\, du = \ln|u| + c$$

$$\boxed{\ln|\sec\theta + \tan\theta| + c}$$

Definite Integration using U-substitution

$$\int \cos(3\theta) - 1 \, d\theta$$

$$\int \cos(3\theta)\, d\theta - \int 1 \, d\theta$$

$u = 3\theta$

$du = 3\, d\theta$

$\frac{1}{3} du = d\theta$

$$\frac{1}{3}\int \cos(u)\, du \quad - \int 1 \, d\theta$$

$$\frac{1}{3}\sin(u) \quad - \theta \quad + C$$

$$\boxed{\frac{1}{3}\sin(3\theta) - \theta + C}$$

$$\int \cot\left(\frac{\theta}{3}\right) d\theta$$

$u = \frac{1}{3}\theta$

$du = \frac{1}{3} d\theta$

$3\, du = d\theta$

$$3\int \cot(u)\, du$$

$$-3 \ln \left| \csc\left(\frac{\theta}{3}\right)\right| + C \quad \text{or} \quad 3\ln\left|\sin\left(\frac{\theta}{3}\right)\right| + C$$