

ABCALC The Definite Integral Homework Solutions

1. Graphically speaking, if $f(x)$ is always above the x-axis, what does $\int_a^b f(x)dx$ mean?

The area between $f(x)$ and the x-axis from a to b .

2. Given the graph of $f(x)$ below, answer the following questions.

- a) Is $\int_a^b f(x)dx$ positive, negative, or zero? Why?

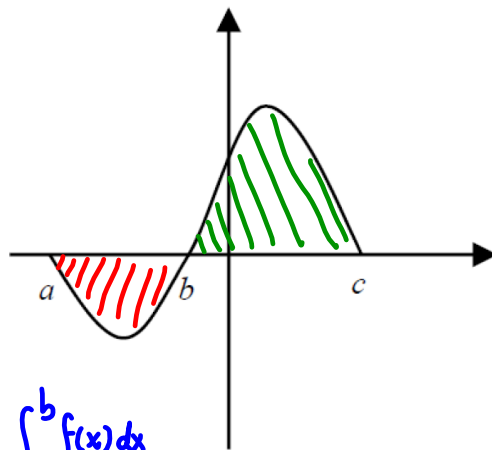
negative since $f(x)$ is below the x-axis from a to b .

- b) Is $\int_b^c f(x)dx$ positive, negative, or zero? Why?

positive since $f(x)$ is above the x-axis from b to c .

- c) Is $\int_a^c f(x)dx$ positive, negative, or zero? Why?

positive since $\int_b^c f(x)dx$ is larger than $\int_a^b f(x)dx$ and $\int_a^c f(x)dx$ is the sum of those two.



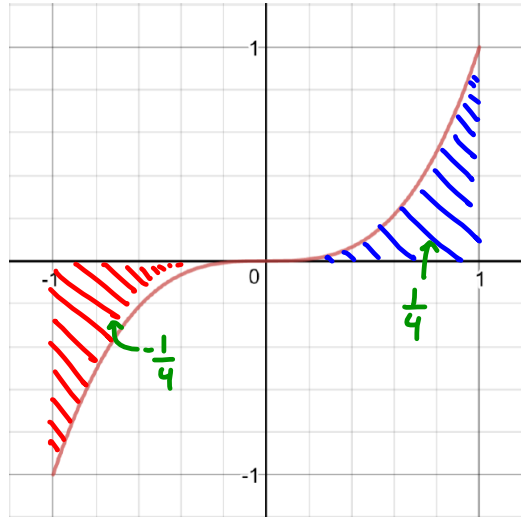
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3. The graph of $y = x^3$ is given below. Use it and the fact that $\int_0^1 x^3 dx = \frac{1}{4}$ to evaluate each of the following.

a) $\int_{-1}^1 x^3 dx = -\frac{1}{4} + \frac{1}{4} = \boxed{0}$

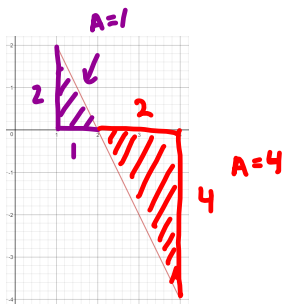
b) $\int_0^1 (x^3 + 3) dx = \int_0^1 x^3 dx + \int_0^1 3 dx = \frac{1}{4} + 3(1) = \boxed{\frac{13}{4}}$

c) $\int_0^1 (x^3 - 1) dx = \int_0^1 x^3 dx - \int_0^1 1 dx = \frac{1}{4} - 1 = \boxed{-\frac{3}{4}}$



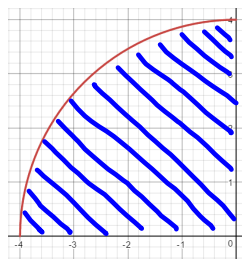
4. Draw a sketch and shade the area indicated by each integral, then use geometry to evaluate the integral.

a) $\int_1^4 (-2x + 4) dx = -3$



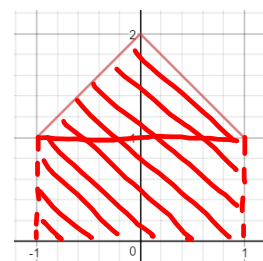
$1 + -4 = \boxed{-3}$

b) $\int_{-4}^0 \sqrt{16 - x^2} dx = 4\pi$



$A = \frac{1}{4} \pi r^2 = \frac{1}{4} \pi (4)^2 = \boxed{4\pi}$

c) $\int_{-1}^1 (2 - |x|) dx = 3$



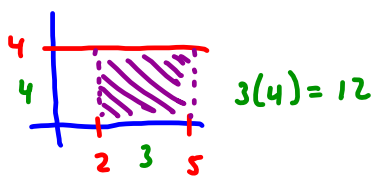
rectangle area = 2
 Δ area = 1
 $2 + 1 = 3$

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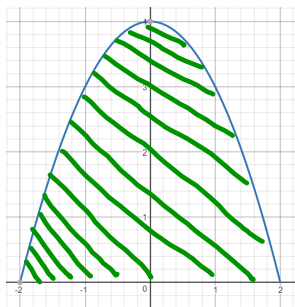
5. If $\int_2^4 f(x)dx = 18$, then find $\int_2^5 (f(x) + 4)dx$

$$\int_2^5 f(x)dx + \int_2^5 4dx$$

$$18 + 12 = \boxed{30}$$



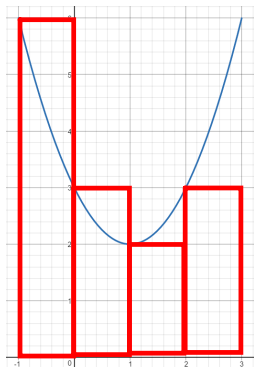
6. Draw a sketch for the area enclosed between the x-axis and the graph of $y = 4 - x^2$ over $[-2, 2]$. Set up an integral to find the area of the region and use your calculator to evaluate the integral.



$$\int_{-2}^2 (4 - x^2)dx = \frac{32}{3}$$

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7. Approximate the area under the curve defined by $y = x^2 - 2x + 3$ from $[-1, 3]$ using the left rectangular approximation method with 4 subintervals of equal length.



$$6 + 3 + 2 + 3 = \boxed{14}$$

8. The function f is continuous over the closed interval $[0, 10]$ and has values that are given in the table.

x	0	2	5	7	10
$f(x)$	2	3	5	7	8

Using 4 subintervals, find each of the following approximations for the area under the curve from $[0, 10]$.

a) LRAM $2(2) + 3(3) + 2(5) + 3(7) = 4 + 9 + 10 + 21 = \boxed{44}$

b) RRAM $2(3) + 3(5) + 2(7) + 3(8) = 6 + 15 + 14 + 24 = \boxed{59}$

c) Trapezoid $\frac{1}{2}(2)(2+3) + \frac{1}{2}(3)(3+5) + \frac{1}{2}(2)(5+7) + \frac{1}{2}(3)(7+8) = \boxed{51.5}$
 $5 + 12 + 12 + 22.5$