AB Calculus Estimating with Finite Sums Notesheet   Name: _____
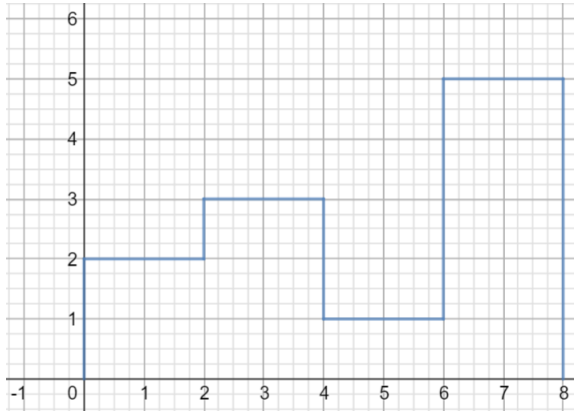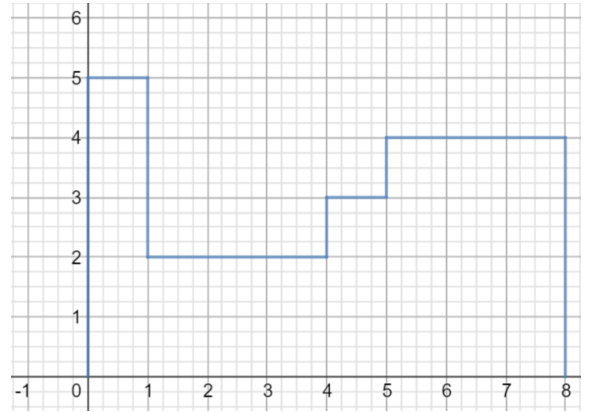
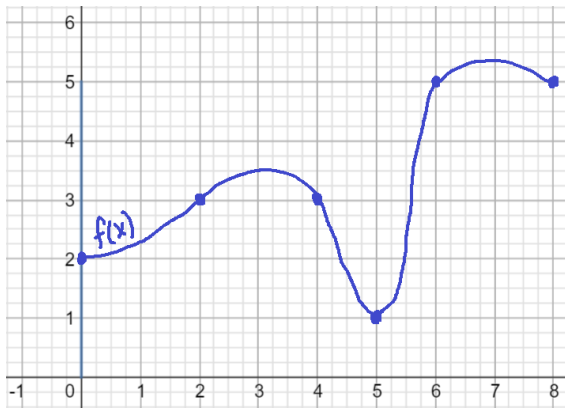**To Begin: Find the area of the region enclosed by the graph of f(x) over the interval [0, 8].**
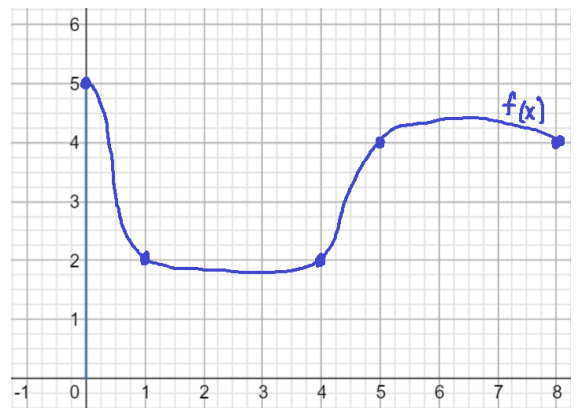

Area: _____

Area: _____

While considering the process you took to find the area of the two regions above, try to **approximate** the area under the following curves and above the x-axis over the interval [0, 8].


Area: _____

Area: _____

Rectangular Approximation Method (RAM) a.k.a. Riemann Sums

Suppose we wanted to know the area of the region bounded by a curve, the x-axis, and the lines $x = a$ and $x = b$, as shown at the right.

The first step is to divide the interval from a to b into subintervals, we can then draw rectangles using the width of each subinterval as the base.

The height of each rectangle is determined by the function value at a point in the specific subinterval, and can be determined using three different methods.
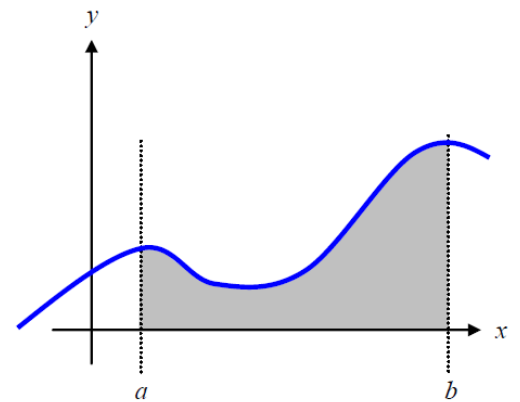
We could use the left endpoint of each subinterval (called **LRAM**), the right endpoint of each subinterval (called **RRAM**), or the midpoint of each subinterval (called **MRAM**).

Example 1: Which rectangular approximation method is shown in the two graphs below?
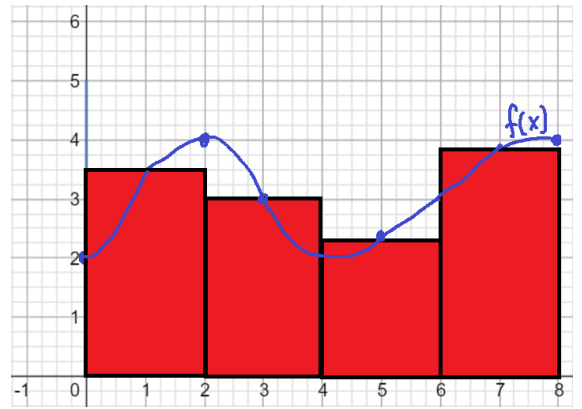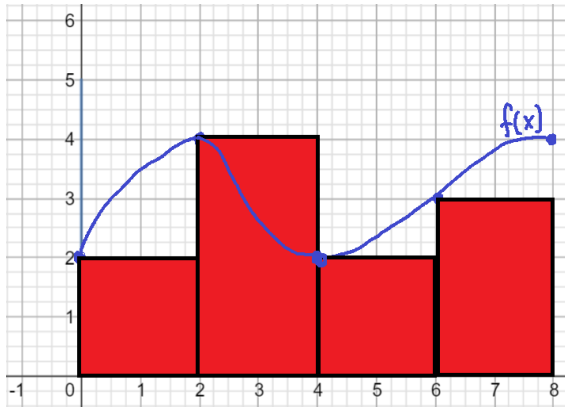
Example 2: The total area under the curve then is approximately equal to the total area of all the rectangles. Which of the graphs above gives a better approximation of the area under the curve? Why? How could it be further improved?

Summary of the Process

Step 1: Divide (or partition) the interval into n subintervals
**Step 2**: Create n rectangles whose base equals the width of the subinterval and whose height is determined by the function value at the left endpoint, the right endpoint, or the midpoint of the subinterval.
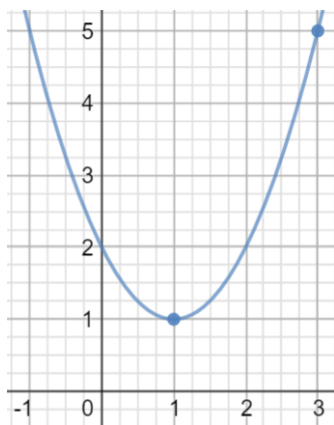**Step 3**: Find the area of all n rectangles and add them together.

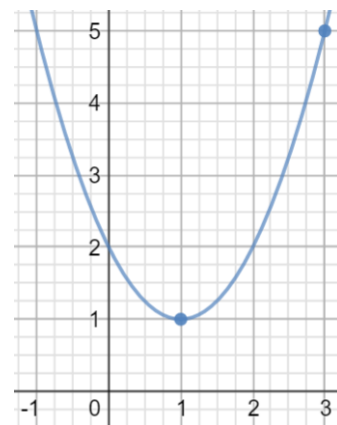**Example 3:** Illustrate the use of RRAM and LRAM on the graphs below. Use 5 rectangles.
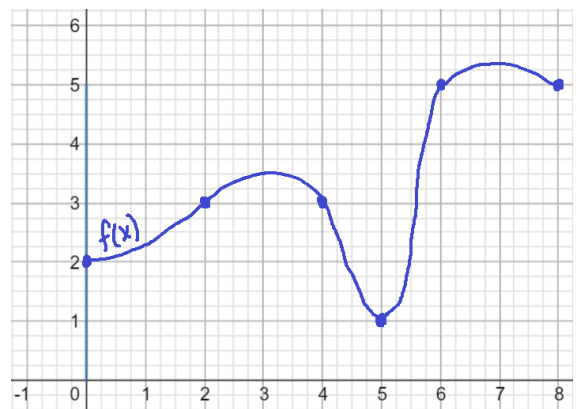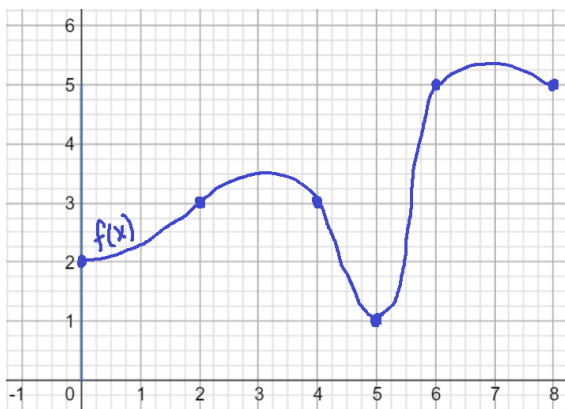
Example 4: Use 4 rectangles to approximate the area under the graph of $y = x^2 - 2x + 2$ from x = 1 to x = 3. Use LRAM, RRAM, and MRAM.
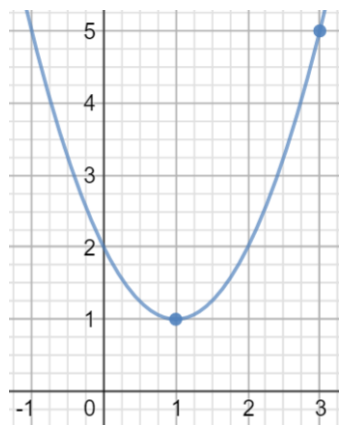
LRAM

RRAM

MRAM

Using your rectangles as a guide, find each approximation.

LRAM =	
RRAM =	
MRAM =	

Example 5: It is not necessary to have a graph to estimate the area. Suppose the table below shows the velocity of a model train engine moving along a track for 10 seconds.

Time (sec)	0	1	2	3	4	5	6	7	8	9	10
Velocity (cm/sec)	0	12	22	10	5	13	11	6	2	6	0

a) Using LRAM with 10 subintervals, estimate the distance traveled by the engine in the first 10 seconds.

LRAM =	

b) Using MRAM with 5 subintervals, estimate the distance traveled by the engine in the first 10 seconds.

MRAM =	

Trapezoid Rule

While rectangles make a fairly good approximation, it's easy to see that we're going to need a lot of them to provide a good estimate. We can find a better estimate in less time if we use trapezoids. If we were to partition the interval into subintervals like we did before, we can use each subinterval to create a trapezoid if we just connect the function values of the left and right endpoints. Before we begin, let's make sure you understand the area formula for a trapezoid.

$$A = \frac{1}{2}h(b_1 + b_2)$$

While not all trapezoids must look like this, the one's we are going to be using will, so we will stick with this picture. Label all the parts of the area formula on the picture below.
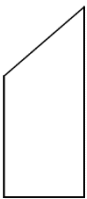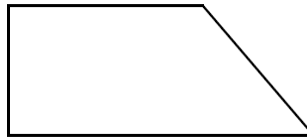
The biggest difference will be the orientation of the trapezoid. The ones we are going to be drawing will look like the trapezoid to the left. The "height" of the trapezoid is just the width of the subinterval, and the bases are going to be the function values of the left and right endpoints.

Example 6: Use 4 trapezoids to approximate the area under the curve $y = x^2 - 2x + 2$ from x = 1 to x = 3. Sketch the trapezoids.
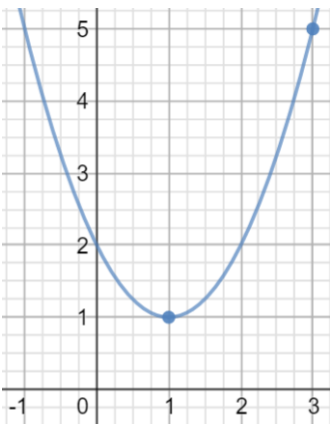
Using Method 1: Find the area of each trapezoid and add together.

Using Method 2 (Works only if all subintervals have the same width):