ABCALC Optimization Day 2 Homework Solutions

1. A poster is to contain 50 square inches of printed matter with margins of 4 inches at both the top and the bottom and 2 inches at each side. Find the dimensions that will minimize the total area of the poster.

$xy = 50$ $\qquad y = \frac{50}{x}$

$A = (x+4)(y+8)$

$A = (x+4)\left(\frac{50}{x}+8\right)$

$A = 50 + 8x + \frac{200}{x} + 32$

$A' = 8 - \frac{200}{x^2}$

$0 = 8 - \frac{200}{x^2}$

$\frac{200}{x^2} = 8$

$8x^2 = 200$

$x^2 = 25$

$x = \pm 5 \qquad x = 5$

A': $-$ | $+$ at 5; A: $\searrow$ 5 $\nearrow$

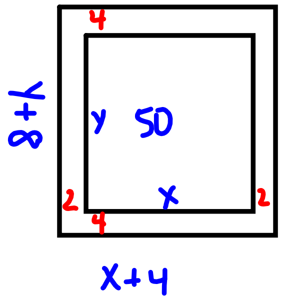
min at $x = 5$

Since A' changes $-$ to $+$

$y = \frac{50}{5} = 10$

$y + 8 = 10 + 8 = 18$

$x + 4 = 5 + 4 = 9$



The dimensions should be 18in by 9in to minimize poster area

2. A rectangular, open-top box is to be made from a piece of cardboard 24 inches long and 9 inches wide by cutting out identical squares from the corners and turning up the cardboard to form the sides. Find the maximum volume of the box.

$0 < x < 4.5$

$V = x(24-2x)(9-2x)$

$V = 216x - 48x^2 - 18x^2 + 4x^3$

$V = 216x - 66x^2 + 4x^3$

$V' = 216 - 132x + 12x^2$

$0 = 18 - 11x + x^2$

$0 = (x-9)(x-2)$

$\cancel{x = 9}$ too big $\qquad x = 2$

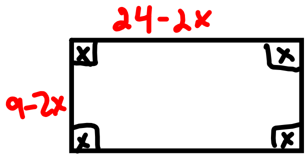
V': $+$ | $-$ at 2; V: $\nearrow$ 2 $\searrow$

max at $x = 2$

Since V' changes $+$ to $-$

$V = 2(24-2(2))(9-2(2))$

$V = 200$



The maximum volume is 200 in^3

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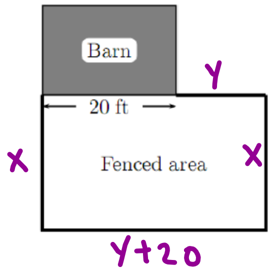
3. A homeowner wishes to erect a fence enclosing a rectangular area adjacent to a barn which is 20 feet long. The diagram below illustrates his plan for the fenced area. Find the largest area that can be enclosed if 96 feet of fencing material is available.



$A = x(y+20)$ $\quad 96 = 2x + 2y + 20$

$76 - 2x = 2y \quad y = 38 - x$

$A = x(38 - x + 20) \quad 2x = 58 \quad y = 38 - 29 = 9$

$A = x(58 - x) \quad x = 29$

$A = 58x - x^2$

A': $+$ | $-$

A: $\nearrow$ 29 $\searrow$

$A' = 58 - 2x$

$0 = 58 - 2x$

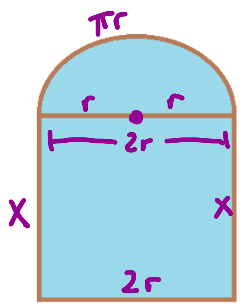
max at $x=29$ since A' changes $+$ to $-$

$A = 29(9+20)$

$A = 29 \cdot 29 = 841$

$\boxed{\text{Max area is 841 sq.ft}}$

4. An arch top window is being built whose bottom is a rectangle and the top is a semicircle. If there is 12 meters of framing materials, find the width of the window that lets in the most light (largest area).



A= Semicircle + rectangle

$A = \frac{1}{2}\pi r^2 + 2rx$

$12 = \pi r + 2x + 2r + 2r$

$12 = \pi r + 2x + 4r \quad x = 6 - \frac{\pi}{2}r - 2r$

$12 - \pi r - 4r = 2x$

$A = \frac{1}{2}\pi r^2 + 2r\left(6 - \frac{\pi}{2}r - 2r\right)$

$A = \frac{1}{2}\pi r^2 + 12r - \pi r^2 - 4r^2$

$A' = \pi r + 12 - 2\pi r - 8r$

$0 = 12 - \pi r - 8r$

$r = 1.077$

A': $+$ | $-$

A: $\nearrow$ 1.077 $\searrow$

max since A' changes $+$ to $-$

width $= 2r$

$2(1.077)$

$\boxed{\text{width} = 2.154\text{ m}}$

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5. Let $f(x) = 12x^{\frac{2}{3}} - 4x$

a) Find the intervals on which f is increasing. Justify your answer.

increasing over $(0, 8)$ since $f' > 0$.

$f' = 8x^{-\frac{1}{3}} - 4$

$0 = \frac{8}{\sqrt[3]{x}} - 4$ und. at $x = 0$

$4 = \frac{8}{\sqrt[3]{x}}$ $\quad 4\sqrt[3]{x} = 8$

$\sqrt[3]{x} = 2$

$x = 8$

b) Find the x-coordinates of all relative maximum and minimum points. Justify your answer.

max at $x = 8$ since f' changes $+$ to $-$

min at $x = 0$ since f' changes $-$ to $+$

f': $-$ | $+$ | $-$

f: $\searrow$ 0 $\nearrow$ 8 $\searrow$

$f(0) = 0$

f is continuous at $x = 0$.

5. Let $f(x) = 12x^{\frac{2}{3}} - 4x$

c) Find the intervals on which f is concave down. Justify your answer.

$f' = 8x^{-\frac{1}{3}} - 4$

$f'' = -\frac{8}{3}x^{-\frac{4}{3}}$

$0 = \frac{-8}{3\sqrt[3]{x^4}}$

$-8 \neq 0$ never $= 0$

$3\sqrt[3]{x^4} = 0$

$x = 0$

f'': $-$ | $-$

f: $\cap$ 0 $\cap$

f is concave down over $(-\infty, 0) \cup (0, \infty)$ since $f'' < 0$.