1. Find two positive numbers such that the sum of the first and twice the second is 100 and their product is a maximum.
2. If 40 passengers hire a special car on a train, they will be charged $\$ 8$ each. This fare will be reduced by $\$ 0.10$ for each passenger, if the number of passengers is over 40 . What number of passengers will produce the most revenue for the railroad?
3. An athletic field is to be built in the shape of a rectangle $x$ units long capped by semicircular regions of radius $r$ at the two ends. The field is to be bounded by a 400-m running track. What values of $x$ and $r$ will give the rectangle the largest possible area?
4. An offshore well is located in the ocean at a point $W$ which is six miles from the closes shore point $A$ on a straight shoreline. The oil is to be piped to a shore point $B$ that is eight miles from $A$ by piping it on a straight line under water from $W$ to some shore point $P$ between $A$ and $B$ and then on to $B$ via a pipe along the shoreline. If the cost of laying pipe is $\$ 100,000$ per mile underwater and $\$ 75,000$ per mile over land, how far from $A$ should the point $P$ be located to minimize the cost of laying the pipe? What will the cost be?

5. A function $f$ is continuous on the closed interval $[-3,3]$ such that $f(-3)=4$ and $f(3)=1$. The functions $f^{\prime}$ and $f^{\prime \prime}$ have the properties given in the table below.

| $x$ | $-3<x<-1$ | $x=-1$ | $-1<x<1$ | $x=1$ | $1<x<3$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $f^{\prime}(x)$ | positive | does not exist | negative | 0 | negative |
| $f^{\prime \prime}(x)$ | positive | does not exist | positive | 0 | negative |

a) What are the $x$-coordinates of all relative maximum and minimum points of $f$ on the interval $(-3,3)$ ? Justify your answer.
b) What are the $x$-coordinates of all points of inflection of $f$ on the interval $[-3,3]$ ? Justify your answer.
c) For what values of $x$ is the graph concave down? Justify your answer.
d) On the axes provided, sketch a graph that satisfies the given conditions of $f$. Answers may vary slightly.


