ABCALC Optimization Day 3 Homework Solutions

1. Find two positive numbers such that the sum of the first and twice the second is 100 and their product is a maximum.

$P = xy \qquad x + 2y = 100 \qquad x = 100 - 2y$

$P = (100-2y)y$

$P = 100y - 2y^2$

$P' = 100 - 4y$ never und.

$0 = 100 - 4y$

$4y = 100$

$y = 25$

P': $+$ | $-$

P: $\nearrow$ 25 $\searrow$

max since P' changes $+$ to $-$.

$x = 100 - 2(25)$

$x = 50$

$\boxed{\text{The two numbers are 50 and 25}}$

2. If 40 passengers hire a special car on a train, they will be charged \$8 each. This fare will be reduced by \$0.10 for each passenger, if the number of passengers is over 40. What number of passengers will produce the most revenue for the railroad? x = number of additional passengers

Revenue = (Passengers)(Price)

$R = (40 + x)(8 - 0.1x)$

$R = 320 - 4x + 8x - 0.1x^2$

$R = 320 + 4x - 0.1x^2$

$R' = 4 - 0.2x$ never und.

$0 = 4 - 0.2x$

$.2x = 4$

$x = 20$

R': $+$ | $-$

R: $\nearrow$ 20 $\searrow$

max at $x = 20$ since R' changes $+$ to $-$

Passengers $= 40 + 20 = 60$

$\boxed{\text{60 passengers will produce the most revenue for the railroad}}$

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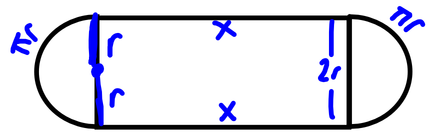
3. An athletic field is to be built in the shape of a rectangle x units long capped by semicircular regions of radius r at the two ends. The field is to be bounded by a 400-m running track. What values of x and r will give the rectangle the largest possible area?

$400 = 2x + 2\pi r$

$400 - 2\pi r = 2x$

$x = 200 - \pi r$



$A = 2rx$

$A = 2r(200 - \pi r)$

$A = 400r - 2\pi r^2$

$A' = 400 - 4\pi r$ never und.

$0 = 400 - 4\pi r$

$4\pi r = 400$

$r = \frac{100}{\pi}$

A′: + | − at $\frac{100}{\pi}$; A ↗ $\frac{100}{\pi}$ ↘

max at $r = \frac{100}{\pi}$ since A′ changes + to −.

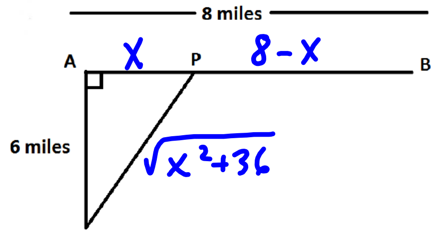
$x = 200 - \pi\left(\frac{100}{\pi}\right)$

$x = 200 - 100 = 100$

$\boxed{\text{x should be 100m and r should be } \frac{100}{\pi} \text{ to maximize the area of the rectangle}}$

4. An offshore well is located in the ocean at a point W which is six miles from the closes shore point A on a straight shoreline. The oil is to be piped to a shore point B that is eight miles from A by piping it on a straight line under water from W to some shore point P between A and B and then on to B via a pipe along the shoreline. If the cost of laying pipe is \$100,000 per mile underwater and \$75,000 per mile over land, how far from A should the point P be located to minimize the cost of laying the pipe? What will the cost be?

$0 < x < 8$



C = underwater + over land

$C = 100000\sqrt{x^2+36} + 75000(8-x)$

$C = 100000\sqrt{x^2+36} + 600000 - 75000x$

$C' = 50000(x^2+36)^{-\frac{1}{2}}(2x) - 75000$ never und.

$0 = 50000(x^2+36)^{-\frac{1}{2}}(2x) - 75000$ CALC.

$x \approx 6.803$

C′: − | + at 6.803; C ↘ 6.803 ↗

min since C′ changes − to +

P should be 6.803 mi downshore from A to Minimize cost at \$996862.70

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5. A function f is continuous on the closed interval $[-3, 3]$ such that $f(-3) = 4$ and $f(3) = 1$. The functions f' and f'' have the properties given in the table below.

x	$-3 < x < -1$	$x = -1$	$-1 < x < 1$	$x = 1$	$1 < x < 3$
$f'(x)$	positive	does not exist	negative	0	negative
$f''(x)$	positive	does not exist	positive	0	negative

a) What are the x-coordinates of all relative maximum and minimum points of f on the interval $(-3, 3)$? Justify your answer.

Relative max at $x=-1$ since f' changes $+$ to $-$.

No Relative min since f' never changes from $-$ to $+$.

5. A function f is continuous on the closed interval $[-3, 3]$ such that $f(-3) = 4$ and $f(3) = 1$. The functions f' and f'' have the properties given in the table below.

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$f'(x)$	positive	does not exist	negative	0	negative
$f''(x)$	positive	does not exist	positive	0	negative

b) What are the x-coordinates of all points of inflection of f on the interval [-3, 3]? Justify your answer.

Point of inflection at $x=1$ since f'' changes sign

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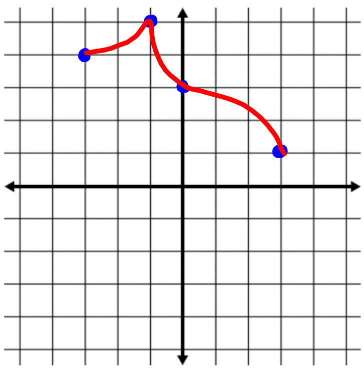
x	$-3 < x < -1$	$x = -1$	$-1 < x < 1$	$x = 1$	$1 < x < 3$
$f'(x)$	positive	does not exist	negative	0	negative
$f''(x)$	positive	does not exist	positive	0	negative

c) For what values of x is the graph concave down? Justify your answer.

f is concave down over $(1,3)$ since $f''<0$.

d) On the axes provided, sketch a graph that satisfies the given conditions of f. Answers may vary slightly.



$f(-3)=4$ $f(3)=1$

Increasing, concave up over $(-3,-1)$

decreasing, concave up $(-1,0)$

POI $x=0$

decreasing, concave down over $(0,3)$