

Good afternoon Happy Monday!

Agenda:

- Return Quizzes → Have HW out
- Remind Code: 19calc20
- Delta Math: 196729

-You need to have your AP College board login ready on Wednesday.

-Continue Limits

Pre-Calc Quiz  
Return  
Find the limits

a)  $\lim_{x \rightarrow -8} f(x) = 2$

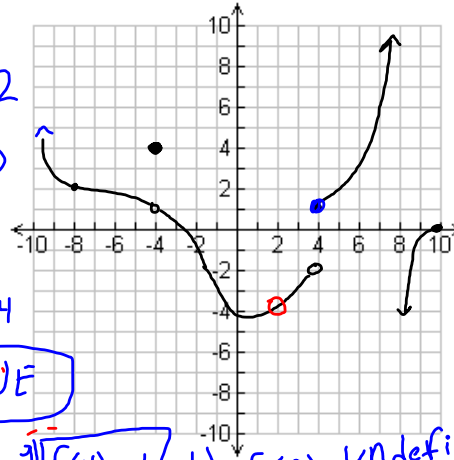
b)  $\lim_{x \rightarrow 4} f(x) = 1$

c)  $f(-4) = 4$

d)  $\lim_{x \rightarrow 2} f(x) = -4$

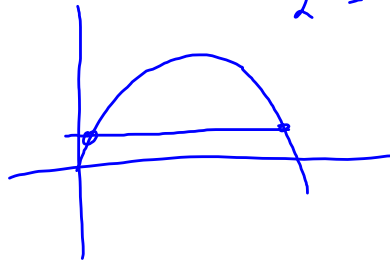
e)  $\lim_{x \rightarrow 4} f(x) = \text{DNE}$

f)  $\lim_{x \rightarrow 8} f(x) = \text{DNE}$ , g)  $f(4) = 1$ , h)  $f(8) = \text{Undefined}$

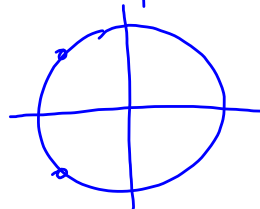
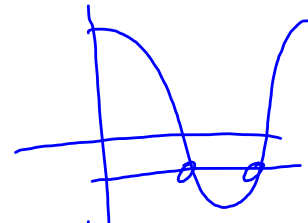


$$e^{\frac{1}{2} \ln 2} = e^{(\ln 2)^{\frac{1}{2}}}$$

$$2^{\frac{1}{2}} = \sqrt{2}$$



$$\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$$



$$\frac{\cot^2 x \cdot \cos^2 x}{\cot^2 x \cdot \cos^2 x}$$

$$\frac{\cos^2 x}{\sin^2 x} \cdot \cos^2 x = \frac{\cos^4 x}{\sin^2 x}$$

$$\frac{\cos^2 x}{\sin^2 x} - (\cos^2 x) \frac{\sin^2 x}{\sin^2 x} = \frac{\cos^2 x - \cos^2 \sin^2 x}{\sin^2 x}$$

$$\frac{\cos^4 x}{\cos^2(1-\sin^2 x)} = \frac{\cos^4 x}{\cos^4 x}$$

⑨  $\tan^2 x - 1 = 0$  (1)  
 $\sin x - 1 = 0$   $\tan^2 x = 1$   
 $\sin x = 1$   $\tan x = \pm 1$

$\left(\frac{\pi}{2}\right)$

$\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$   
 $\sqrt{x^2 - 3 + 2}$

⑩

$\sqrt{x+2} + 2$

$\sqrt{x^2 - 1}$

$\sqrt{(x+1)(x-1)}$

⑪

H.A.  
 $y = \frac{1}{2}$

V.A.  
 $x = \frac{1}{2}$

Hole  
 $x = -1$

$\frac{(x+1)(x-1)}{(2x-1)(x+1)}$

$2^4 = (2^3)^{3x}$

$2^4 = 2^{9x}$

$\frac{4}{9} = x$

$(2x+3)(3x-1)$

$x = -\frac{3}{2} \quad x = \frac{1}{3}$

$$X = \frac{\ln(y-5)^3}{3}$$

$$3x = \ln(y-5)^3 \quad \therefore \sqrt[3]{3x} = \ln(y-5)$$

$$3x = 3 \ln(y-5)$$

$$x = \ln(y-5)$$

$$e^x = y-5$$

$$e^x + 5 = y$$

$$e^{\sqrt[3]{3x}} = y-5$$
$$(e^{3x})^{\frac{1}{3}} = y-5$$
$$e^x + 5 = y$$

$$\cot^2(x) \cos^2(x)$$

$$\cot^2(x) \cdot \cos^2(x) \cdot \frac{\sin^2(x)}{\sin^2(x)}$$

$$\frac{\cos^2(x)}{\sin^2(x)} \cdot \cos^2(x) = \frac{\cos^4(x)}{\cancel{\sin^2(x)}}$$

$$\frac{\cos^2 x}{\sin^2 x} - \frac{\cos^2 x \sin^2 x}{\sin^2 x} = \frac{\cos^2 x (1 - \sin^2(x))}{\cancel{\sin^2 x}} = 1$$

$$\tan^2 x - 1 = 0$$

$$\sin x - 1 = 0$$

$$\sin x = 1$$

$$\left(\frac{\pi}{2}\right)$$

$$\tan^2 x = 1$$

$$\tan x = \pm 1$$

$$\frac{\pi}{4}, \left(\frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\right)$$

$$\sqrt{x^2 - 3 + 2}$$

$$a) \sqrt{\sqrt{x+2} + 2}$$

$$HA \quad VA \\ y = \frac{1}{2} \quad x = \frac{1}{2}$$

$$2^4 = (2^3)^{3x}$$

$$2^4 = 2^{9x} \\ x = \frac{4}{9}$$

$$x = \frac{\ln(y-5)^3}{3}$$

$$3x = \ln(y-5)^3$$

$$3x = 3 \ln(y-5)$$

$$x = \ln(y-5)$$

$$e^x = y-5$$

$$e^x + 5 = y$$

$$b) \sqrt{x^2 - 1}$$

$$\sqrt{(x+1)(x-1)}$$

$$\frac{(x+1)(x-1)}{(2x-1)(x+1)}$$

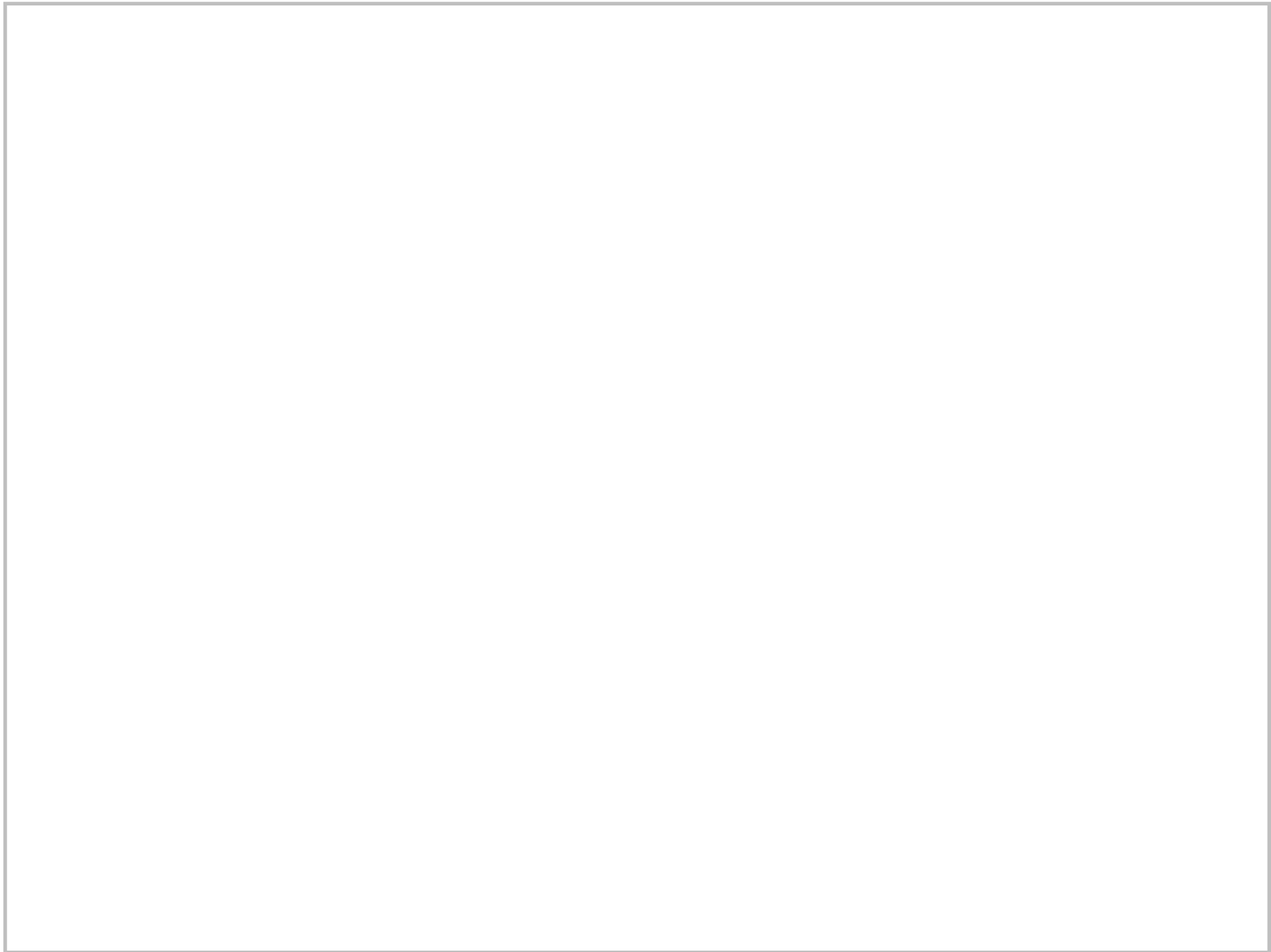
$$(2x+3)(3x-1)$$

$$x = \frac{-3}{2} \quad x = \frac{1}{3}$$

$$e^{\sqrt[3]{3x}} = \ln(y-5)$$

$$e^{(3x)^{1/3}} = y-5$$

$$e^x + 5 = y$$



# Limits:

1) Why do you think limits for piece wise function are commonly taken as  $x$  approaches the break?

$$f(x) = \begin{cases} x^2 + 3 & x < 0 \\ x + 3 & x \geq 0 \end{cases}$$

$$\lim_{x \rightarrow 0} f(x)$$



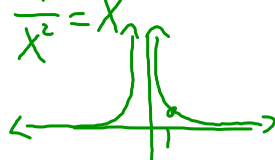
## Limits that fail to exist

i. Different left and right behavior

$$\lim_{x \rightarrow 0} \frac{|x|}{x} \quad \begin{array}{l} |x| \leq x, x > 0 \\ |x| \geq x, x < 0 \end{array}$$
$$\boxed{\text{DNE}} \quad \frac{|x|}{x} \begin{array}{l} \leq 1, x > 0 \\ \geq -1, x < 0 \end{array}$$

ii. Unbounded Behavior

$$f(x) = \frac{1}{x^2} = x^{-2} \quad \lim_{x \rightarrow 0} \frac{1}{x^2} \text{ DNE}$$



iii. Oscillating Behavior

$$f(x) = \sin\left(\frac{1}{x}\right) \quad \text{DNE}$$

$$\lim_{x \rightarrow 0} \sin\left(\frac{1}{x}\right)$$



$$\lim_{x \rightarrow c} f(x) = f(c) \quad \text{Continuous @ } c$$

Thm 1.1 (1.2)

Let  $b$  and  $c$  be real numbers, and let  $n$  be a positive integer. (let  $f$  and  $g$  be functions with limits;  $\lim_{x \rightarrow c} f(x) = L$  and  $\lim_{x \rightarrow c} g(x) = k$ .)

Th 1.1

$$\lim_{x \rightarrow c} b = b \quad \lim_{x \rightarrow c} x = c$$

$$\lim_{x \rightarrow c} x^n = c^n \quad (\text{Direct substitution})$$

(1.2)

Scalar multiple  $\lim_{x \rightarrow c} b \cdot f(x) = b \lim_{x \rightarrow c} f(x) = b \cdot L$

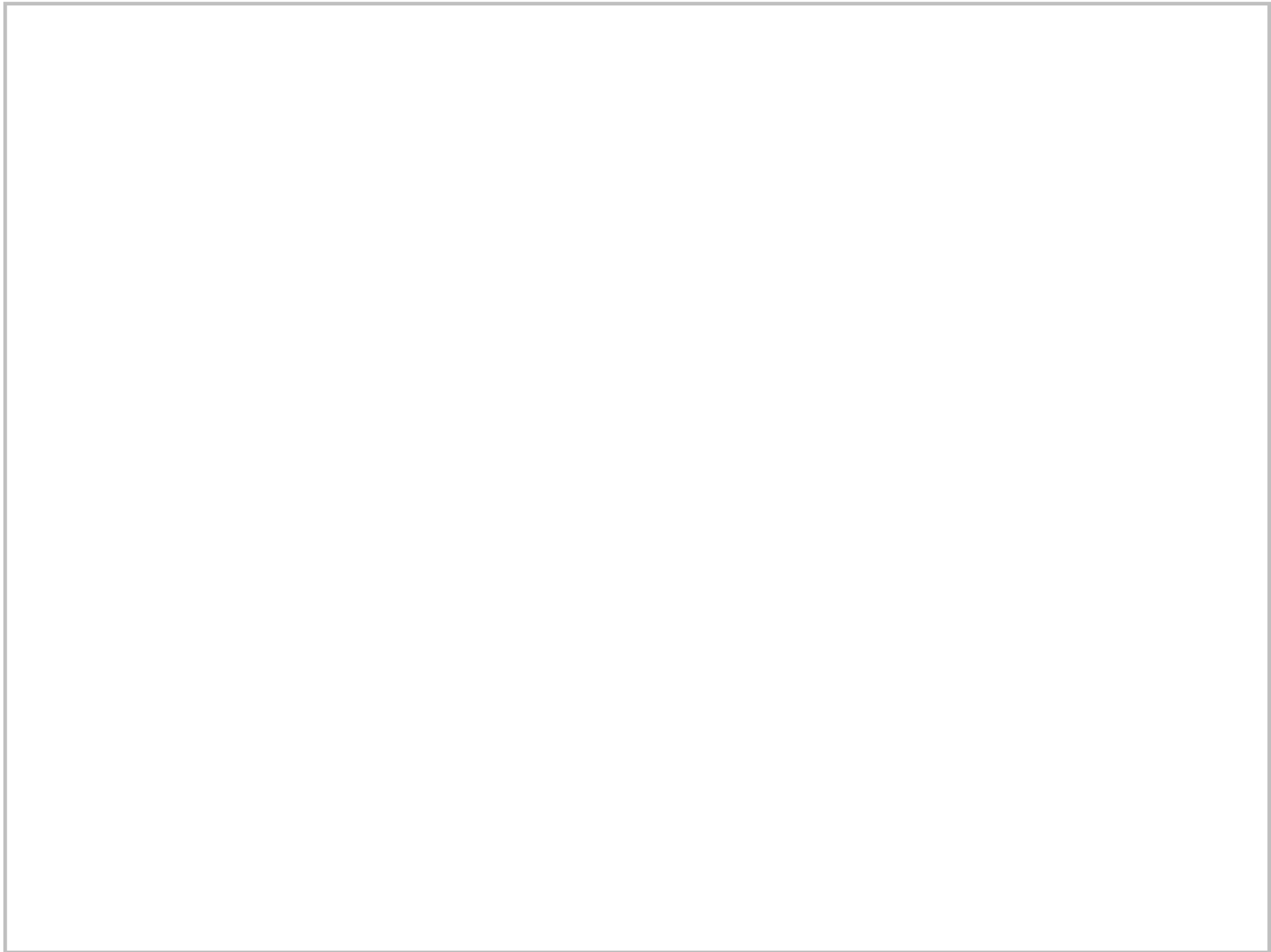
sum or difference  $\lim_{x \rightarrow c} [f(x) \pm g(x)] = \lim_{x \rightarrow c} f(x) \pm \lim_{x \rightarrow c} g(x) = L \pm K$

Product  $\lim_{x \rightarrow c} (f(x) \cdot g(x))$

$$\lim_{x \rightarrow c} f(x) \cdot \lim_{x \rightarrow c} g(x) = L \cdot k$$

Quotient  $\lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow c} f(x)}{\lim_{x \rightarrow c} g(x)} = \frac{L}{k}$

Power  $\lim_{x \rightarrow c} [f(x)]^n = \left[ \lim_{x \rightarrow c} f(x) \right]^n = L^n$   $k \neq 0$



$$\lim_{x \rightarrow c} f(x) = f(c)$$

Continuous  $y=x$ ,  $y=x^2$   
polynomial  
 $3x^2+7x+2$

Thm

1.)

Let  $b$  and  $c$  be a real number and let  $n$

be a positive integer  $f(x)=x$

$$\lim_{x \rightarrow c} b = b \quad \lim_{x \rightarrow c} x = c$$

$$\lim_{x \rightarrow c} x^n = c^n$$

EX]  $\lim_{x \rightarrow 2} 3 = 3 \quad \lim_{x \rightarrow -4} x = -4$

$$\lim_{x \rightarrow 2} x^4 = 2^4 = 16$$

① scalar  $\lim_{x \rightarrow c} b \cdot f(x) = b \cdot L$   
 $b \lim_{x \rightarrow c} f(x)$

② sum or difference  $\lim_{x \rightarrow c} [f(x) \pm g(x)]$   
 $\lim_{x \rightarrow c} f(x) \pm \lim_{x \rightarrow c} g(x)$   
 $= L \pm K$

③ Product

$$\lim_{x \rightarrow c} (f(x) \cdot g(x)) = \lim_{x \rightarrow c} f(x) \cdot \lim_{x \rightarrow c} g(x)$$
$$= L \cdot K$$

(4) Quotient

$$\lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow c} f(x)}{\lim_{x \rightarrow c} g(x)}$$

$$= \frac{L}{k} \quad k \neq 0$$

Power

$$\lim_{x \rightarrow c} [f(x)]^n = \left[ \lim_{x \rightarrow c} f(x) \right]^n$$
$$= L^n$$

$$\lim_{x \rightarrow 2} 4x^2 + 3 = \lim_{x \rightarrow 2} 4x^2 + \lim_{x \rightarrow 2} 3$$

Direct

$$4(2)^2 + 3$$

(19)

$$4 \lim_{x \rightarrow 2} x^2 + \lim_{x \rightarrow 2} 3$$
$$4(4) + 3$$
$$16 + 3 = 19$$

1.3] EX]  $\lim_{x \rightarrow 1} \frac{x^2 + x + 2}{x + 1} = \frac{\lim_{x \rightarrow 1} x^2 + x + 2}{\lim_{x \rightarrow 1} x + 1}$

( $x \neq -1$ )

$$\frac{4}{2} = 2$$

$$\sqrt{x}, x \geq 0$$

# Limits Properties and Analytically.

