Good afternoon Happy Friday!

Agenda:

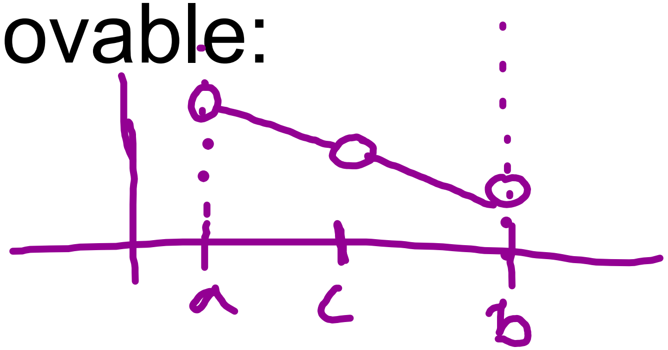
-Quiz

-One sided limits and continuity

2 Types of discontinuities @ a Point

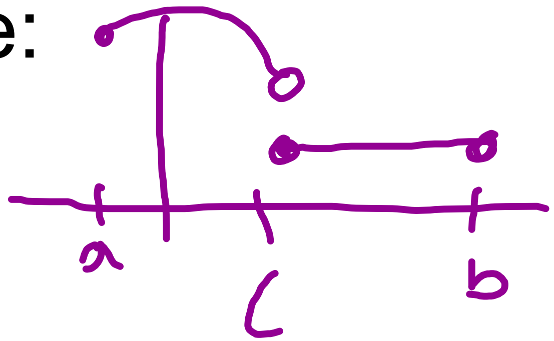
1) Removable:

- Hole:

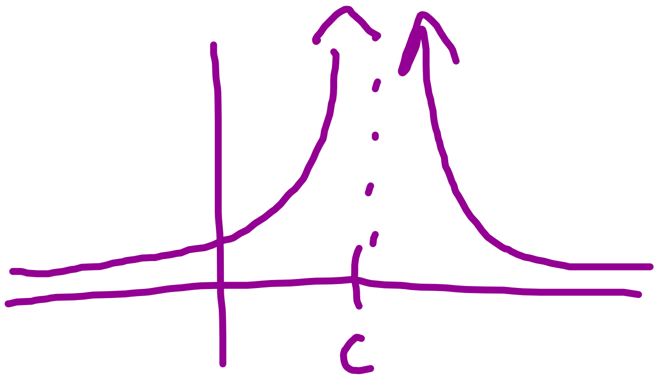


2) Non-Removable:

- Jump



- Infinite



Rationalizing to find the limit:

$$\lim_{x \to 0} \frac{4-\sqrt{16-x}}{x} \cdot \frac{4+\sqrt{16-x}}{4+\sqrt{16-x}}$$

$$\lim_{x \to 0} \frac{16-\left(\sqrt{16-x}\right)^2}{x\left(4+\sqrt{16-x}\right)} = \lim_{x \to 0} \frac{16-(16-x)}{x\left(4+\sqrt{16-x}\right)}$$

$$\lim_{x \to 0} \frac{\cancel{x}}{\cancel{x}\left(4+\sqrt{16-x}\right)} = \lim_{x \to 0} \frac{1}{4+\sqrt{16-x}}$$

$$\frac{1}{8}$$

$$16-(16-x)$$

$$16-16+x$$

3 conditions apply for continuity at a point $x=c$

1) $f(c)$ is defined

2) $\lim_{x \to c} f(x)$ exists

3) $\lim_{x \to c} f(x) = f(c)$

If any one is not met then it is discontinuous at the point x=c.

Continuity on an open interval.

A function is continuous on open interval (a,b) when the function is continuous at each point in (a,b). A function that is continuous on the entire real number line $(-\infty, \infty)$ is "everywhere continuous"

Discuss the continuity of each function

One-sided limits and continuity on a closed interval.

Limit from the $\boxed{\text{right}}$

$$\lim_{x \to c^{+}} f(x) = L$$

Limit from the Left

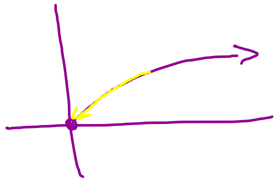
$$\lim_{x \to c^{-}} f(x) = L$$

$$\lim_{x \to -2^{-}} f(x)$$

Radical

n - even

$$\lim_{x \to 0^{+}} \sqrt{x} = 0$$



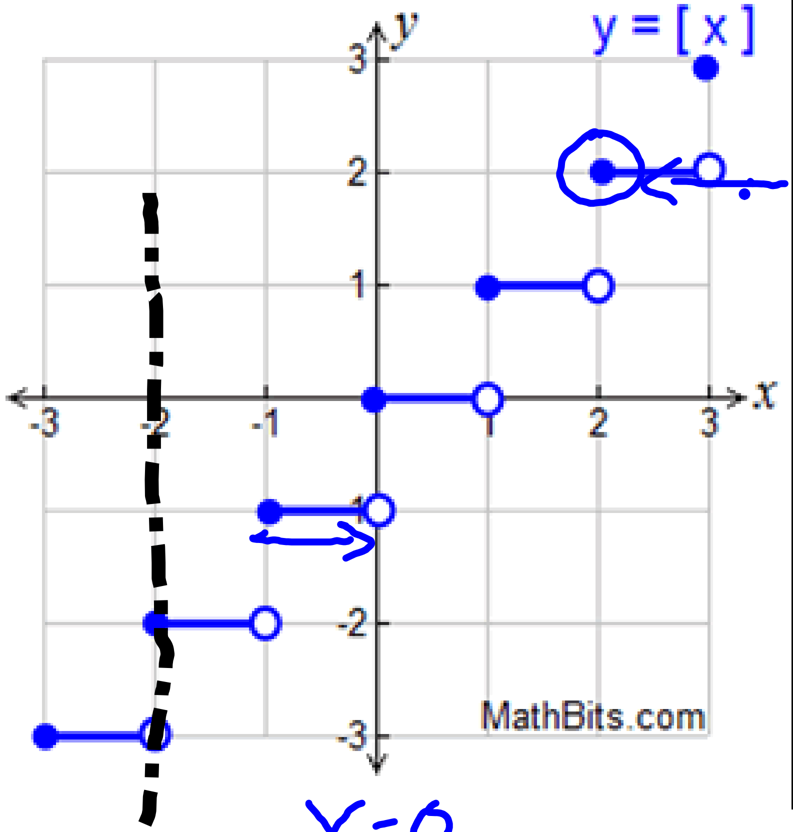
$$\lim_{x \to 0^{-}} \sqrt{x} = DNE$$

Example $f(x)= \sqrt{(4-x^2)}$

$$\lim_{x \to -2^+} f(x) = 0$$

Example 2: $f(x) = \boxed{[x]}$

$$\lim_{x \to 0^-} f(x) = -1$$



x=0

$$\lim_{x \to 2^+} f(x)$$

$$\lim_{x \to -2^-} f(x) = -3$$

The existence of a Limit

Let f be a function and let c and L be real numbers. The limit of f(c) as x approaches c is L if and only if

$$\lim_{x \to c^-} f(x) = L = \lim_{x \to c^+} f(x,$$

A function is continuous

$[a, b]$;

1st f is continuous on (a, b)

2nd

$\lim_{x \to a^+} f(x) = f(a)$ and $\lim_{x \to b^-} f(x) = f(b)$

Properties of continuity:

If b is a real number and f and g are continuous at x=c, then the functions listed below are also continuous at c.

Continuity of a composite function

if g is continuous at c and f is continuous at g(c), then f(g(x)) is continuous at c.

Testing for continuity:

Solve for values that make the function continuous.

Generally piece wise functions.