

Good afternoon Happy Thursday!

Agenda:

-30 minutes Pre-Calc Review quiz

-Intermediate value theorem

-Infinite limits and Limits at infinity

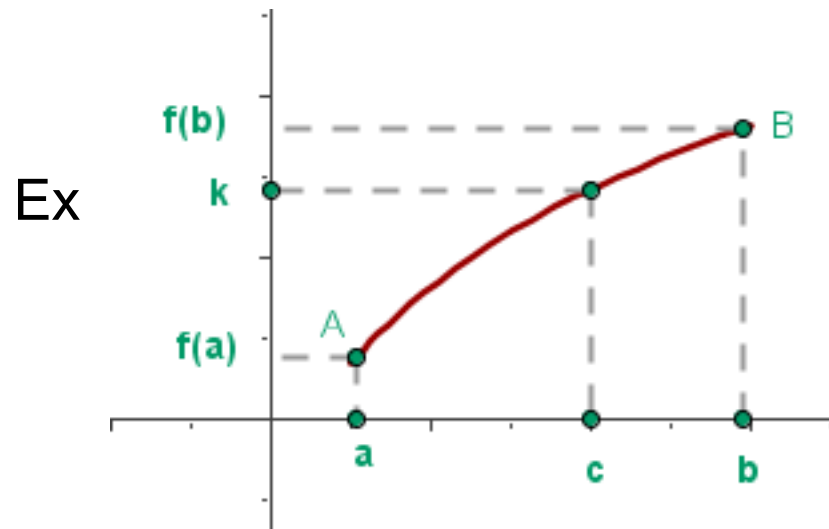
-Squeeze Theorem

Intermediate Value Theorem(1st real theorem)

If f is continuous on $[a,b]$, $f(a) \neq f(b)$, and k is any number between $f(a)$ and $f(b)$, then

there is at least one number c in $[a,b]$ such that $f(c) = k$.

** this is an existence theorem**



Application of IVT

$$f(x) = x^3 + 2x - 1$$

show it has a zero
(root)

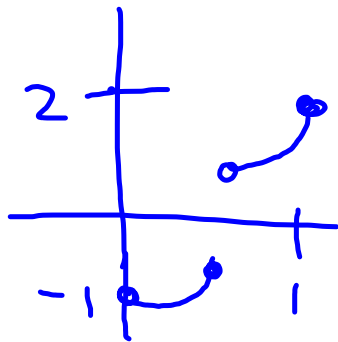
$$[0, 1]$$

$$f(0) = -1$$

$$f(1) = 2$$

$$-1 < 0 < 2$$

$$f(0) < 0 < f(1)$$



Infinite limits:
What is an infinite limit?

Consider $y=3/(x-2)$

H.A. $y=0$

V.A. @ $x=2$

$$\lim_{x \rightarrow 2} \frac{3}{x-2} \Rightarrow \frac{3}{1.9-2} = \frac{3}{-.1} = -30$$

$$\lim_{x \rightarrow 2} \frac{3}{x-2} = -\infty \quad \frac{3}{1.99-2} = \frac{3}{-.01} = -300$$

$$\frac{3}{1.999-2} = \frac{3}{-.001} = -3000$$

$$\lim_{x \rightarrow 2^+} \frac{3}{x-2} = \infty$$

Note that our answer $\lim f(x) = \infty$ does not mean the limit exists! In fact it tells how it fails to exist, by representing the unbounded behavior of $f(x)$ at c .

Properties:

$$\lim_{x \rightarrow c} f(x) = \infty$$

$$\lim_{x \rightarrow c} g(x) = L$$

$$f \pm g = \infty \quad f \cdot g = \infty \quad L > 0$$
$$= -\infty \quad L < 0$$

$$\frac{g}{f} = 0 \quad \frac{1}{\infty} = 0$$

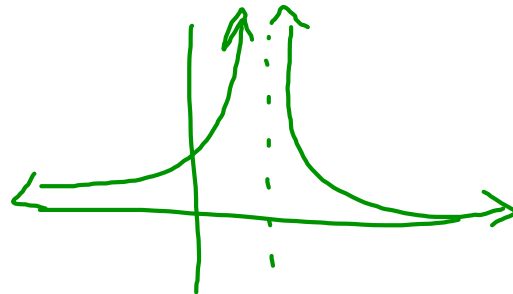
Vertical Asymptote:

IF $f(x)$ approaches infinity (negative infinity) as x approaches c , from the right or left, then the line $x=c$ is a vertical asymptote of the graph f .

EX) $\frac{1}{(x-1)^2} \cdot \frac{1}{.00001}$

$$\lim_{x \rightarrow 1^-} \frac{1}{(x-1)^2} = \infty$$

$$\lim_{x \rightarrow 1^+} \frac{1}{(x-1)^2} = \infty$$



| : -

EX)

$$\lim_{x \rightarrow 2^-} \tan \frac{\pi x}{4}$$

∞

$$\lim_{x \rightarrow 2^+} \tan \frac{\pi x}{4}$$

$-\infty$

Limits at infinity:

Consider: $f(x) = \frac{3x^2}{x^2+1}$

$$y = 3$$

$$\lim_{x \rightarrow \infty} \frac{3x^2}{x^2+1} = \frac{\infty}{\infty} \text{ indeterminate}$$

$$\lim_{x \rightarrow \infty} \frac{3}{1 + \frac{1}{x^2}} = \frac{\lim_{x \rightarrow \infty} 3}{\lim_{x \rightarrow \infty} 1 + \lim_{x \rightarrow \infty} \frac{1}{x^2}} = \frac{3}{1+0}$$

$\frac{1}{(10000)^2} = 0.00000001$ $\frac{3}{1} = \boxed{3}$

$$\lim_{x \rightarrow -\infty} \frac{3x^2}{x^2+1} = \frac{\lim_{x \rightarrow -\infty} 3}{\lim_{x \rightarrow -\infty} 1 + \lim_{x \rightarrow -\infty} \frac{1}{x^2}}$$

$\frac{3}{1} = \boxed{3}$ \downarrow
C

$$\lim_{x \rightarrow \infty} \frac{C}{x^r} = 0$$

$$\lim_{x \rightarrow -\infty} \frac{C}{x^r} = 0$$

x^r defined
when $x < 0$

$$\lim_{x \rightarrow \infty} e^{-x} \rightarrow \frac{1}{e^x} = 0$$

$$\lim_{x \rightarrow -\infty} e^x = 0$$

Horizontal Asymptote:

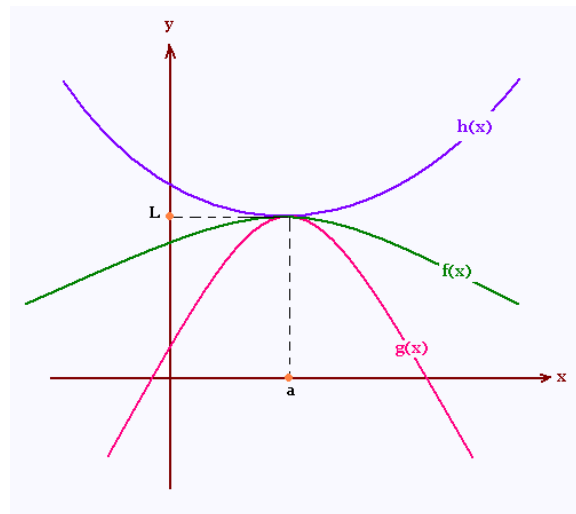
The line $y=L$ is a horizontal asymptote of the graph f when

$$\lim_{x \rightarrow \infty} f(x) = L$$

Squeeze theorem:

If $h(x) \leq f(x) \leq g(x)$ for all x in an open interval containing c , except possibly at c , and if

then



3 special cases:

1)

2)

3)

