Good Morning Happy Monday!
Happy First day of Fall!!
Agenda:
-Warm UP
-Squeeze Theorem
-Answer Question from Homework on Infinite limits and Limits at infinity

-Pre-Calc Re-Quiz will be returned once everyone who plans on taking it completes it.

-Quiz Wednesday, Test on Limits Friday!

Warm UP:

1) Describe the difference between an infinite limit and a limit at infinity.

2) Solve the following limits

ⓐ $\lim_{x \to 2^+} \sec\left(\frac{3\pi x}{4}\right)$

check a with graph

ⓑ $\lim_{x \to \infty} \frac{\sqrt{4x^5+4}}{3x^2-1}$

$\frac{\lim_{x \to \infty} \sqrt{4x}}{3} = \infty$

3) What is a real world example of the Intermediate value theorem? unbounded

if $f(x)$ is continuous $[a,b]$

$f(a) \neq f(b)$, $f(a) \leq K \leq f(b)$,

then c in $[a,b]$

$f(c) = K$.

$$y = \frac{x^2-1}{2x+4} \qquad \text{V.A. } x=-2$$

$$\boxed{\lim_{x \to -2^-}} \frac{x^2-1}{2x+4} \to \frac{(-2.0001)^2 - 1}{2(-2.0001)+4}$$

$$\frac{\sim 3}{\text{very small neg. \#}} \Rightarrow \frac{\sim 3}{-0.00001} = \boxed{-\infty}$$

$$\boxed{\lim_{x \to -2^+} \frac{x^2-1}{2x+4} \quad \infty} = \frac{(-1.999)^2-1}{2(-1.999)+4} = \frac{\sim 3}{\text{very small (+) \#}} = \frac{\sim 3}{0.0001}$$

$$\cos x = 0$$

$$y = \sec x \to \frac{1}{\cos x} \qquad \frac{\pi}{2} \quad \frac{3\pi}{2}$$

$$\lim_{x \to \frac{\pi}{2}^-} \frac{1}{\cos x} \Rightarrow \frac{1}{0.0001} = \infty$$

$$\lim_{x \to \frac{\pi}{2}^+} \frac{1}{\cos x} \Rightarrow \frac{1}{-0.0001} = -\infty$$

$$y=\left(2-\frac{x}{x+1}\right)\left(\frac{x^2}{5+x^2}\right)$$

$$\lim_{x\to\infty}\left(2-\frac{\frac{x}{x}}{\frac{x}{x}+\frac{1}{x}}\right)\cdot\lim_{x\to\infty}\frac{\frac{x^2}{x^2}}{\frac{5}{x^2}+\frac{x^2}{x^2}}$$

$$\left(2-\frac{1}{1+0}\right)\cdot\frac{1}{0+1}$$

$$(2-1)\cdot 1 = 1\cdot 1=\boxed{1}$$

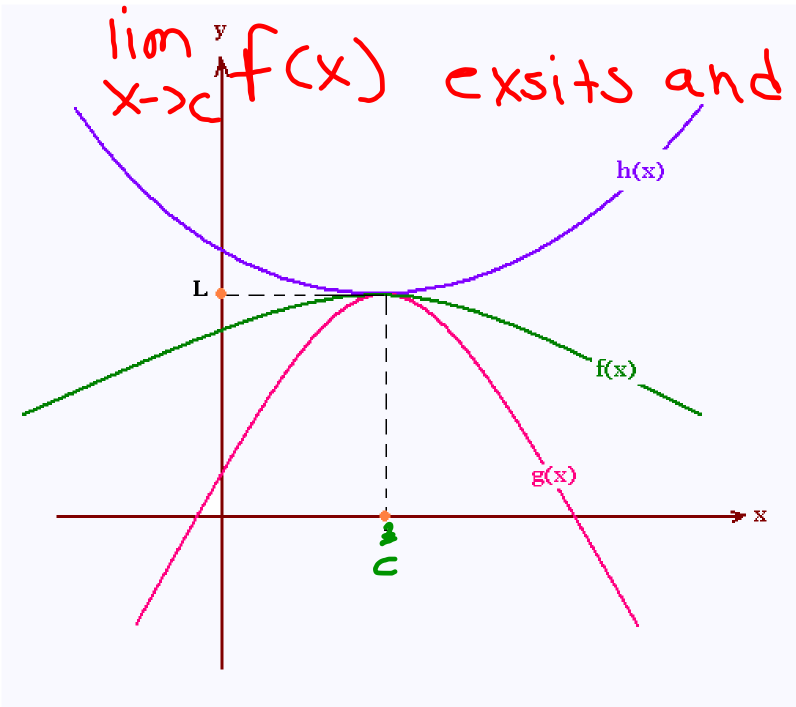
Squeeze theorem:

If $h(x) \le f(x) \le g(x)$ for all x in an open interval containing c, except possibly at c, and if

$$\lim_{x \to c} h(x) = L = \lim_{x \to c} g(x)$$

then

$\lim_{x \to c} f(x)$ exsits and is L.



Special cases(Trig) :

1) $$\boxed{\lim_{x\to 0} \frac{\sin x}{x} = 1}$$

2) $$\boxed{\lim_{x\to 0} \frac{1-\cos x}{x}} \cdot \frac{1+\cos x}{1+\cos x} =$$

$$\lim_{x\to 0} \frac{1-\cos^2 x}{x(1+\cos x)} \Rightarrow \lim_{x\to 0} \frac{\sin^2 x}{x(1+\cos x)}$$

$$\underbrace{\lim_{x\to 0} \frac{\sin x}{x}}_{1} \cdot \lim_{x\to 0} \frac{\sin x}{(1+\cos x)}$$

$$\frac{0}{2}$$

$$1 \cdot 0 = \boxed{0}$$

$$\lim_{x\to 0} \frac{\tan x}{x} \Rightarrow \frac{\frac{\sin x}{\cos x}}{x}$$

$$\lim_{x\to 0} \frac{\sin x}{\cos x} \cdot \frac{1}{x}$$

$$* \lim_{x\to 0} \frac{\sin x}{x} \cdot \lim_{x\to 0} \frac{1}{\cos x} \quad \frac{1}{\cos(0)}$$

$$1 \cdot 1 = \boxed{1}$$

$$\lim_{x\to 0} \frac{4}{4} \frac{\sin(4x)}{x} \Rightarrow \lim_{x\to 0} \frac{4\sin(4x)}{4x}$$

$$4\left(\lim_{x\to 0} \frac{\sin(4x)}{4x}\right) = 4 \cdot 1 = 4$$

b.f

$$\lim_{x\to 0} \frac{\tan(2x)}{6x \to 3\cdot 2x} \Rightarrow \frac{\tan(2x)}{3\cdot 2x}$$

$$\frac{1}{3} \lim_{x\to 0} \frac{\tan(2x)}{2x}$$

$$\frac{1}{3} \cdot 1 = \boxed{\frac{1}{3}}$$

You try:

$$\cos(x) \le g(x) \le x^2+1$$

$$\lim_{x\to 0} \sin\left(\frac{1}{x}\right) = DNE$$

$$-1 \le \sin\left(\frac{1}{x}\right) \le 1$$

$$-1x^2 \le \boxed{x^2 \sin\left(\frac{1}{x}\right)} \le x^2$$

$$\lim_{x\to 0} -x^2 = 0 \qquad \boxed{\lim_{x\to 0} x^2 \sin\left(\frac{1}{x}\right) = \boxed{0}} \qquad \lim_{x\to 0} x^2 = 0$$

Limits at infinity finish

Horizontal Asymptote:

The line y=L is a horizontal asymptote of the graph f when

$$\lim_{x\to -\infty} f(x) = L \quad \text{or} \quad \lim_{x\to \infty} f(x) = L$$

$$y = \frac{x^2-1}{2x+4}$$

$$\lim_{x \to -2} \frac{x^2-1}{2x+4}$$

$$\lim_{x \to -2^+} \frac{(-1.999)^2-1}{2(-1.9999)+4} \to \frac{+}{-3.9999+4}$$

$$\frac{\sim 3}{+0.00001} = \infty$$

$$\lim_{x \to -2^-} \frac{x^2-1}{2x+4} \Rightarrow \frac{(-2.0001)^2-1}{2(-2.0001)+4} = \frac{\sim 3}{-0.0001}$$

$$-\infty = \frac{\sim 3}{\text{small neg.}}$$

$$\lim_{x \to \infty} \frac{2x+\sin x}{x}$$

$$\lim_{x \to \infty} \frac{2\cancel{x}/x}{x/x} + \lim_{x \to \infty} \frac{\sin x}{x}$$

$$2 + 0$$

$$= \boxed{2}$$

$$\lim_{x \to \infty} \sin x \; \frac{1}{x} = 0$$

$$-\frac{1}{x} \le \frac{\sin x}{x} \le \frac{1}{x}$$

Title: Sep 23-11:08 AM (15 of 15)