Lesson Area between Curves:

U-Substitution: https://www.geogebra.org/m/YPSNUfpm

Area b/w curves General: https://www.geogebra.org/m/YCt3bvBE

Area b/w curves Specific: https://www.geogebra.org/m/rdjwwd3z

*Logic Game: https://www.geogebra.org/m/pjPgJdhV

U-Substitution: Recall to solve some integral U-substitution is required. This is considered a "reverse chain rule" its not the only way to solve but it's a way the we have focused on in class.

$\int f\big(g(x)\big)g'(x)dx \qquad u = g(x) \qquad du = g'(x)$ so the integral we are now dealing with is $\int f(u)du$

This should be a simpler function to integrate. If we are find the general or particular solution then we need to "back substitute u" in order for our function to be in terms of the original variable.

Example: $\int 3(8y-1)e^{4y^2-y}dy$ *Yes its in terms of y but it doesn't change the process, feel free to change to x.*

$u = 4y^2 - y$, *it's inside the exponential function* $du = 8y - 1$ *dy (convenient)*

$$3\int e^u\,du = 3e^u + c = \mathbf{3}\boldsymbol{e}^{\mathbf{4}\boldsymbol{y}^{\mathbf{2}}-\boldsymbol{y}} + \boldsymbol{C}$$

**So our general solution is $\mathbf{3}\boldsymbol{e}^{\mathbf{4}\boldsymbol{y}^{\mathbf{2}}-\boldsymbol{y}} + \boldsymbol{C}$**

**A particular solution** relies on some given condition: such as F(0)=3 or $F(1) = 10$ where F(y) is the antiderivative. Basically you use the given condition to solve for the constant C.

Given $F(0) = 16$ solve for the particular solution, $3e^{4(0)^2-0} + C = 16 \rightarrow 3 + C = 16 \rightarrow C = 13$

SO the particular solution is $\mathbf{3}\boldsymbol{e}^{\mathbf{4}\boldsymbol{y}^{\mathbf{2}}-\boldsymbol{y}} + \mathbf{13}$

Changing Bounds as you see in the U-Substitution link to GeoGebra

If solving the definite integral then you can go through the "back substitution" process using same bounds or you can change the bounds using the defined "U" and integrate normally using the Fundamental theorem of calculus.

Example: $\int_{-2}^{0} 2t^2\sqrt{1-4t^3}\,dt$ $\qquad U = 1 - 4t^3 \qquad \boldsymbol{du} = -12t^2 dt$

$2\int_{-2}^{0} t^2\sqrt{1-4t^3}\,dt \qquad 2\left(-\frac{\mathbf{1}}{\mathbf{12}}\right)\int \sqrt{u}\,du \rightarrow$ Antiderivative of $\sqrt{u} = u^{\frac{1}{2}} \rightarrow \frac{1}{\frac{3}{2}}u^{\frac{3}{2}} = \frac{2}{3}u^{\frac{3}{2}}$

***Here is where integrate and make our choice, to change the bounds or keep them the same by "back substituting"***

*Changing the bounds:* $u = 1 - 4t^3 \rightarrow$

$$New\ Lower\ bound: 1 - 4(-2)^3 = 1 - (4 * -8) = 1 - -32 = 33$$

$$New\ Upper\ bound: 1 - 4(0)^3 = 1$$

$-\frac{1}{6}\left(\frac{2}{3}u^{\frac{3}{2}}\right)\ from\ 33\ to\ 1 \rightarrow -\frac{1}{9}\left(u^{\frac{3}{2}}\right) from\ 33\ to\ 1 = -\frac{1}{9} - \left(-\frac{1}{9}(33)^{\frac{3}{2}}\right) =$

If you back substituted, you should get the same result but your function would be

$$-\frac{1}{9}(1-4t^3)^{\frac{3}{2}}\ from - 2\ to\ 0$$