**Examples**

In a card game, suppose a player needs to draw two cards of the same suit in order to win. Of the 52 cards, there are 13 cards in each suit. Suppose first the player draws a heart. Now the player wishes to draw a second heart. Since one heart has already been chosen, there are now 12 hearts remaining in a deck of 51 cards. So the conditional probability *P(Draw second heart|First card a heart)* = 12/51.

Suppose an individual applying to a college determines that he has an 80% chance of being accepted, and he knows that dormitory housing will only be provided for 60% of all of the accepted students. The chance of the student being accepted *and* receiving dormitory housing is defined by
*P(Accepted and Dormitory Housing) =*

*P(Dormitory Housing|Accepted)P(Accepted)*= (0.60)\*(0.80) = 0.48.

**To calculate the probability of the intersection of more than two events, the conditional probabilities of *all* of the preceding events must be considered. In the case of three events, *A*, *B*, and *C*, the probability of the intersection *P(A and B and C) = P(A)P(B|A)P(C|A and B)*.**

Consider the college applicant who has determined that he has 0.80 probability of acceptance and that only 60% of the accepted students will receive dormitory housing. Of the accepted students who receive dormitory housing, 80% will have at least one roommate. The probability of being accepted *and* receiving dormitory housing *and* having no roommates is calculated by:
*P(Accepted and Dormitory Housing and No Roommates) =*

*P(Accepted)P(Dormitory Housing|Accepted)P(No Roomates|Dormitory Housing and Accepted)* =

 (0.80)\*(0.60)\*(0.20) =

0.096. The student has about a 10% chance of receiving a single room at the college.

Another important method for calculating conditional probabilities is given by ***Bayes's formula***.

The formula is based on the expression *P(B) = P(B|A)P(A) + P(B|Ac)P(Ac)*,

 which simply states that the **probability of event *B*** is the sum of the conditional probabilities of event *B* given that event *A* has or has not occurred.

For **independent** events *A* and *B*, this is equal to

*P(B)P(A) + P(B)P(Ac) = P(B)****(P(A) + P(Ac)****) =*

*P(B)(1) = P(B),*since the probability of an event and its complement must always sum to 1. Bayes's formula is defined as follows:



EX: Suppose a voter poll is taken in three states. In state A, 50% of voters support the liberal candidate, in state B, 60% of the voters support the liberal candidate, and in state C, 35% of the voters support the liberal candidate. Of the total population of the three states, 40% live in state A, 25% live in state B, and 35% live in state C. Given that a voter supports the liberal candidate, what is the probability that she lives in state B?

By Bayes's formula:

*P(Voter lives in state B|Voter supports liberal candidate) =*

P(Voter supports liberal candidate|Voter lives in state B)P(Voter lives in state B*)/(P(Voter supports lib. cand.|Voter lives in state A)P(Voter lives in state A) + P(Voter supports lib. cand.|Voter lives in state B)P(Voter lives in state B) + P(Voter supports lib. cand.|Voter lives in state C)P(Voter lives in state C))*

= (0.60)\*(0.25)/((0.50)\*(0.40) + (0.60)\*(0.25) + (0.35)\*(0.35))

= (0.15)/(0.20 + 0.15 + 0.1225) = 0.15/0.4725 = **0.3175.**