Limits and Continuity Practice Test

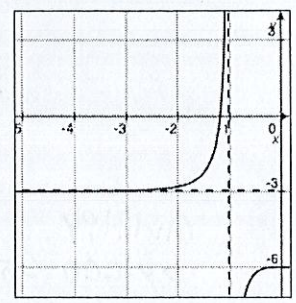
1. Find $\lim_{x \to 0} \frac{6x^5 - 8x^3}{9x^3 - 6x^5}$ $\frac{2x^3(3x^2-4)}{3x^3(3-2x^2)}$ $\quad \frac{2(-4)}{3(3)} = -8/9$

   a. $\frac{2}{3}$
   b. $-\frac{8}{9}$
   c. $\frac{4}{3}$
   d. $-\frac{8}{3}$
   e. Nonexistent

2. $\lim_{x \to -\infty} (5x - 1) =$ DNE
   Unbounded

$\lim_{x \to \pm\infty} \frac{ax^4}{x^4} = -3$ $\quad \boxed{a = -3}$

3. The function f is given by $f(x) = \frac{ax^4+6}{x^4+b}$. The figure to the right shows a portion of the graph of f. Which of the following could be the values of the constants a and b?



   a. $a = -3,\ b = -1$
   b. $a = 3,\ b = 1$
   c. $a = 3,\ b = -1$
   d. $a = 3,\ b = -1$
   e. $a = 6,\ b = -1$

$x = -1$ V.A $\quad (-1)^4 + b = 0$ $\quad \boxed{b = -1}$

4. Find $\lim_{x \to -\infty} \frac{(3x-1)(x^2-4)}{(2x+1)^2(x-1)}$ $\quad \frac{3x^3}{4x^3}$ $\quad 4x^2 \cdot x = 4x^3$

   a. $-\frac{3}{2}$
   b. $\frac{3}{2}$
   c. $\frac{3}{4}$
   d. 1
   e. ∞

5. The functions f and g are continuous. The function h is given by $h(x) = f(g(x)) - x$. The table below gives values of the functions. Explain why there must be a value for t for $1 < t < 4$ such that $h(t) = -1$.

x	1	2	3	4
$f(x)$	0	8	-3	6
$g(x)$	3	7	-1	2

By I.V.T
$h(1) \le h(t) \le h(4)$
$-4 \le -1 \le 4$
there exist t, $1 \le t \le 4$

$h(1) = f(g(1)) - 1 = f(3) - 1 = -3 - 1 = \boxed{-4}$
$h(4)\ f(g(4)) - 4 = f(2) - 4$
$8 - 4$
$= 4$

6. Let $F(x) = \begin{cases} \frac{x^2-5x-6}{x-6}, & x \neq 6 \\ 3k+2, & x = 6 \end{cases}$

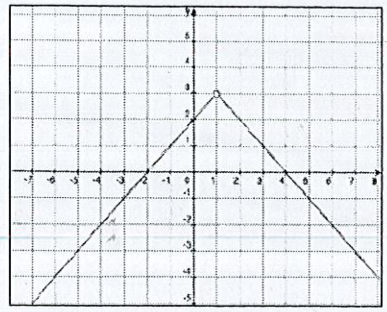
   a. Find $\lim_{x \to 6} F(x)$. Show all proper steps.

   $\frac{\cancel{(x-6)}(x+1)}{\cancel{(x-6)}}$ $\quad \lim_{x \to 6} (x+1) = 7$

   b. Find the value k such that $\lim_{x \to 6} F(x) = F(6)$. Show all work.

   $\lim_{x \to 6} F(x) = 7$ $\quad f(6) = 3k + 2$
   $7 = 3k + 2$ $\quad \frac{7-2}{3} = \boxed{k = \frac{5}{3}}$

7. The figure shows the graph of $f(x)$. Which of the following statements are true?



I. $\lim_{x\to1^-} f(x)$ exists

II. $\lim_{x\to1^+} f(x)$ exists

III. $\lim_{x\to1} f(x)$ exists

a. I only
b. II only
c. I and II only
d. I, II, and III only
e. None are true

8. $\lim_{x\to5}\frac{x}{x-5}=$ DNE

Infinite Discontinuity

$\frac{4.999}{-0.001}\to-\infty$ $\quad \frac{5.001}{0.001}\to\infty$

9. Given $f(x)=\frac{6x+1}{\sqrt{4x^2+6x+9}}$, write an equation for any horizontal asymptote(s) of $f(x)$.

$\frac{6}{\sqrt{4}}=\frac{6}{2}=3$ $\quad \lim_{x\to\pm\infty} f(x)=3$

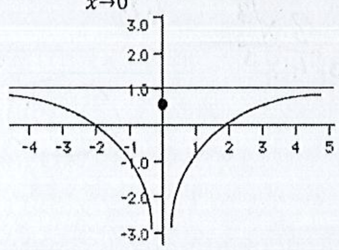
10. Given the function $f(x)=\frac{\frac{1}{x+5}-\frac{1}{5}}{x}$, determine $\lim_{x\to0} f(x)$.

Multiply by LCD $(5(x+5))$

$\lim_{x\to0}\frac{\frac{5(x+5)}{x+5}-\frac{5(x+5)}{5}}{x\cdot5(x+5)}=\frac{5-(x+5)}{x(5(x+5))}=\frac{-x}{x(5(x+5))}=\frac{-1}{5(x+5)}$

$\boxed{\lim_{x\to0} f(x)=\frac{-1}{25}}$

11. For the function $f(x)$ shown below, find $\lim_{x\to0} f(x)$.



$\lim_{x\to0} f(x)$ DNE

12. If $a\neq0$ and n is a positive integer, then $\lim_{x\to a}\frac{x^n-a^n}{x^{2n}-a^{2n}}$ is

$\frac{x^n-a^n}{(x^n-a^n)(x^n+a^n)}=\frac{1}{x^n+a^n}$

a. $\frac{1}{a^n}$
b. $\frac{1}{2a^n}$
c. $\frac{1}{a^{2n}}$
d. 0
e. Nonexistent

$\frac{1}{a^n+a^n}=\frac{1}{2a^n}$

13. What are all the horizontal asymptotes of $f(x)=\frac{6+3e^x}{3-e^x}$ in the xy-plane?

a. $y=3$ only
b. $y=-3$ only
c. $y=2$ only
d. $y=-3$ and $y=0$
e. $y=-3$ and $y=2$

Note

$e^x\cdot e^{-x}=e^{x-x}=e^0=1$

$\lim_{x\to-\infty} f(x)=\frac{6+3e^x}{3-e^x}$ $\quad 3e^x\to0$, $e^x\to0$

$\frac{6}{3}=2$

$\lim_{x\to\infty} f(x)=\frac{(6+3e^x)}{(3-e^x)}\cdot\frac{e^{-x}}{e^{-x}}$

$\lim_{x\to\infty}\frac{6e^{-x}+3}{3e^{-x}-1}=\frac{0+3}{0-1}=\boxed{-3}$

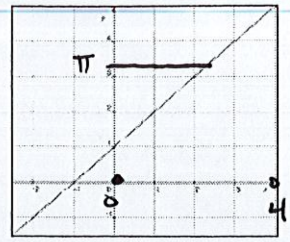
$\lim_{x \to 4^+} 4^2 - 6 = 16 - 6 = 10$

$\lim_{x \to 4^-} 3(4) - 2$

$12 - 2 = 10$

14. Given $f(x) = \begin{cases} x^2 - 6, & x \geq 4 \\ 3x - 2, & x < 4 \end{cases}$, find $\lim_{x \to 4} f(x)$. $= \boxed{10}$

15. The straight-line function f is shown by the graph. Explain why there must be a value x between 0 and 4 such that $f(x) = \pi$.



By: I.V.T.

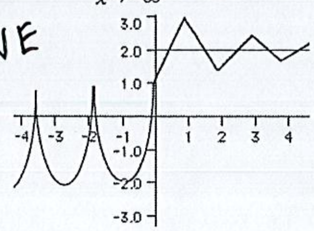
$f(0) \leq f(x) \leq f(4)$

$1 \leq f(x) = \pi \leq 5$

there exist an x such that x is π.

16. For the function $f(x)$ graphed below, find $\lim_{x \to -\infty} f(x)$.

DNE

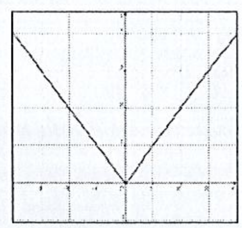


17. The graph of $f(x) = \sqrt{x^2 + 0.0001} - 0.01$ is shown in the graph to the right. Which of the following statements are true?

I. $\lim_{x \to 0} f(x) = 0$

II. f is continuous at $x = 0$

III. $f(0)$ is defined



a. I only
b. II only
c. I and II only
d. I, II and III only
e. None are true.

18. Let $f(x) = \frac{2}{x^2}$ and $g(x) = x^2 - 6$. Find $\lim_{x \to -\infty} f(x) \cdot g(x)$. $= \lim_{x \to -\infty} \dfrac{\frac{2x^2}{x^2} - \frac{12}{x^2}}{\frac{x^2}{x^2}}$ $\to 0$

$\frac{2}{1}$

$\boxed{2} \Leftarrow \frac{2}{1}$

• Limit exists ✓

• $f(0) = 2$ ✓

$\lim_{x \to 0} f(x) = 2$ ✓

19. Let $f(x)$ be given by the function $f(x) = \begin{cases} g(x) + a, & x \leq 0 \\ 3 - b\cos x, & x > 0 \end{cases}$, where a and b are constants and $g(x) = 1 - x^2$. Show that $f(x)$ is continuous at $x = 0$ if $a = 1$ and $b = 1$.

$\lim_{x \to 0^-} f(x) = 1 - (0)^2 + 1 = \boxed{2}$

$\lim_{x \to 0^+} f(x) = 3 - 1\cos(0) = 3 - 1 = \boxed{2}$

20. For the function $f(x)$ shown to the right, find $\lim_{x \to 3} f(x)$.

DNE