Limits and Continuity Practice Test
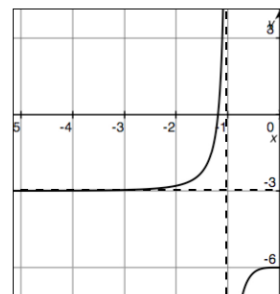
1. Find $\lim\limits_{x \to 0} \dfrac{6x^5 - 8x^3}{9x^3 - 6x^5}$

   a. $\dfrac{2}{3}$

   b. $-\dfrac{8}{9}$

   c. $\dfrac{4}{3}$

   d. $-\dfrac{8}{3}$

   e. Nonexistent

2. $\lim\limits_{x \to -\infty} (5x - 1) =$

3. The function f is given by $f(x) = \dfrac{ax^4 + 6}{x^4 + b}$. The figure to the right shows a portion of the graph of f. Which of the following could be the values of the constants a and b?

   a. $a = -3, \ b = -1$

   b. $a = 3, \ b = 1$

   c. $a = 3, \ b = -1$

   d. $a = 3, \ b = -1$

   e. $a = 6, \ b = -1$

4. Find $\lim\limits_{x \to -\infty} \dfrac{(3x - 1)(x^2 - 4)}{(2x + 1)^2(x - 1)}$

   a. $-\dfrac{3}{2}$

   b. $\dfrac{3}{2}$

   c. $\dfrac{3}{4}$

   d. 1

   e. ∞

5. The functions f and g are continuous. The function h is given by $h(x) = f\big(g(x)\big) - x$. The table below gives values of the functions. Explain why there must be a value for t for $1 < t < 4$ such that $h(t) = -1$.

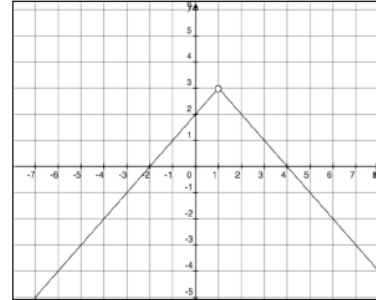 | x | 1 | 2 | 3 | 4 |
   |--------|---|---|----|---|
   | $f(x)$ | 0 | 8 | -3 | 6 |
   | $g(x)$ | 3 | 7 | -1 | 2 |

6. Let $F(x) = \begin{cases} \dfrac{x^2 - 5x - 6}{x - 6}, & x \neq 6 \\ 3k + 2, & x = 6 \end{cases}$

   a. Find $\lim\limits_{x \to 6} F(x)$. Show all proper steps.

   b. Find the value k such that $\lim\limits_{x \to 6} F(x) = F(6)$. Show all work.

7. The figure shows the graph of $f(x)$. Which of the following statements are true?

    I. $\lim\limits_{x\to 1^-} f(x)$ exists

    II. $\lim\limits_{x\to 1^+} f(x)$ exists

    III. $\lim\limits_{x\to 1} f(x)$ exists

    a. I only
    b. II only
    c. I and II only

    d. I, II, and III only
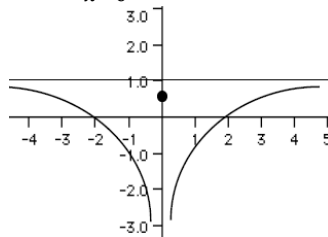   e. None are true

8. $\lim\limits_{x\to 5} \dfrac{x}{x-5} =$

9. Given $f(x) = \dfrac{6x+1}{\sqrt{4x^2+6x+9}}$, write an equation for any horizontal asymptote(s) of $f(x)$.

10. Given the function $f(x) = \dfrac{\frac{1}{x+5} - \frac{1}{5}}{x}$, determine $\lim\limits_{x\to 0} f(x)$.

11. For the function $f(x)$ shown below, find $\lim\limits_{x\to 0} f(x)$.

12. If $a \neq 0$ and n is a positive integer, then $\lim\limits_{x\to a} \dfrac{x^n - a^n}{x^{2n} - a^{2n}}$ is
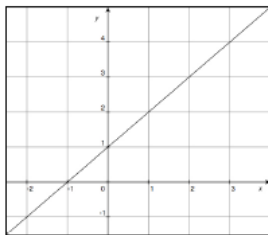
    a. $\dfrac{1}{a^n}$

    b. $\dfrac{1}{2a^n}$

    c. $\dfrac{1}{a^{2n}}$

    d. 0

    e. Nonexistent

13. What are all the horizontal asymptotes of $f(x) = \dfrac{6+3e^x}{3-e^x}$ in the xy −plane?
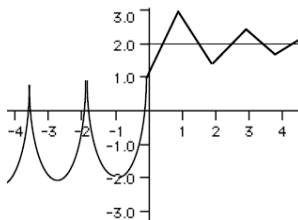
    a. $y = 3$ only
    b. $y = -3$ only
    c. $y = 2$ only

    d. $y = -3$ and $y = 0$
    e. $y = -3$ and $y = 2$

14. Given $f(x) = \begin{cases} x^2 - 6, & x \geq 4 \\ 3x - 2, & x < 4 \end{cases}$, find $\lim_{x \to 4} f(x)$.

15. The straight-line function f is shown by the graph. Explain why there must be a value x between 0 and 4 such that $f(x) = \pi$.
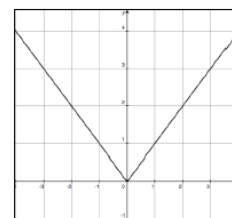

16. For the function $f(x)$ graphed below, find $\lim_{x \to -\infty} f(x)$.

17. The graph of $f(x) = \sqrt{x^2 + 0.0001} - 0.01$ is shown in the graph to the right. Which of the following statements are true?

   I. $\lim_{x \to 0} f(x) = 0$
   II. f is continuous at $x = 0$
   III. $f(0)$ is defined

   a. I only
   b. II only

   c. I and II only
   d. I, II and III only

   e. None are true.

18. Let $f(x) = \frac{2}{x^2}$ and $g(x) = x^2 - 6$. Find $\lim_{x \to -\infty} f(x) \cdot g(x)$.

19. Let $f(x)$ be given by the function $f(x) = \begin{cases} g(x) + a, & x \leq 0 \\ 3 - b \cos x, & x > 0 \end{cases}$, where a and b are constants and $g(x) = 1 - x^2$. Show that $f(x)$ is continuous at $x = 0$ if $a = 1$ and $b = 1$.

20. For the function $f(x)$ shown to the right, find $\lim_{x \to 3} f(x)$.