Test Review:

Solve each equation.

1. $3^{3n} = (1/81)$

$\log_3\left(\frac{1}{81}\right) = 3n$

$-4 = 3n$ $\boxed{n = \frac{-4}{3}}$

2. $10^{3b} = 100000$

$\log(100000) = 3b$

$5 = 3b$ $\boxed{b = \frac{5}{3}}$

3. $-18^p = 98$

$18^p = -98$

$\log_{18}(-98) = p \Rightarrow$ No solution

4. $\frac{3(15^x)}{3} = \frac{6}{3}$

$15^x = 2$

$\log_{15}(2) = x$

$\frac{\log(2)}{\log(15)} = \boxed{x = .25596}$

Solve each equation.

6. $\cancel{\log_{19}}(8-r) = \cancel{\log_{19}}(-3r)$

$8 - r = -3r$

$8 = -2r$

$\boxed{r = -4}$ ✓

7. $\cancel{\log}(3x+2) = \cancel{\log}(2x-1)$

$3x + 2 = 2x - 1$

$x = -3$

No solution log can't have negative inside.

8. $\cancel{\ln}(72 - 2b) = \cancel{\ln}(10b)$

$72 - 2b = 10b$

9. $\log_3(x-3) + \log_3 6 = 5$

$\log_3((x-3)\cdot 6) = 5$

$3^5 = 6x - 18$

$243 = 6x - 18$

$261 = 6x$

$x = 43.5$

10. $2\log_3 y = \log_3 4 + \log_3(y+8)$

$\log_3 y^2 = \log_3(4(y+8))$

$y^2 = 4y + 32$

$y^2 - 4y - 32 = 0$

$(y-8)(y+4)$

$y = 8$ $\quad \cancel{y = -4}$ Can't be negative in log!

Solving Exponential and Logarithmic Equations

Solve each equation. (Note: for #14-17, use the definition of logarithms to change these problems to exponential form.)

11. $\frac{4\log_3 3x}{4} = \frac{20}{4}$

$\log_3(3x) = 5$

$3x = 3^5$

$\boxed{x = 81}$

12. $\log_8(x-5) = 2/3$

$8^{\frac{2}{3}} = x - 5$

$4 = x - 5$

$\boxed{x = 9}$

13. $3\log_5 (x+2) = 6$

$\log_5(x+2) = 2$

$x+2 = 5^2$
$\quad -2 \quad\; -2$

$x = 25 - 2$

$\boxed{x = 23}$

14. $4\ln x = 5$

$\ln(x) = \frac{5}{4}$

$e^{\frac{5}{4}} = x$

$\boxed{3.4903 = x}$

Applications:

Some will solve for $A, p, k\ or\ t$

A) **You have 63 grams of cobalt 60 (half life = 5.27 years). How many grams will there be after 3 years?**

$31.5 = 63e^{k \cdot 5.27}$

$\frac{\ln\left(\frac{1}{2}\right)}{5.27} = \frac{5.27k}{5.27}$

$k = -.1315$

$A = 63e^{-.1315 \cdot 3}$

≈ 42.46 grams

B) **You measure 37 grams of strontium 90 (half life = 28.8 years). How much was present 2 years before this?**

$\frac{1}{2}$ of start $\rightarrow 18.5 = 37e^{28.8 \cdot k}$

$\frac{1}{2} = e^{28.8k}$

$\ln\left(\frac{1}{2}\right) = 28.8k$

$\frac{\ln\left(\frac{1}{2}\right)}{28.8} = k = -.0241$

past $\Rightarrow$ negative time

$A = 37e^{-0.0241 \cdot -2}$

≈ 38.82 grams

$0.00\,0\,4.36$ ~~E-4~~

$\times 10^{-4}$

.000-456

C) After 500 years, a sample of radium-226 has decayed to 80.4% of its original mass. Find the half-life of radium-226. $t = ?$

80.4% of 10

$P = 10$

$\frac{8.04}{10} = \frac{10 e^{r 500}}{10}$

$.804 = e^{500 \cdot r}$

$\frac{\ln(.804)}{500} = \frac{500r}{500}$

$.000436 = r$

$5 = 10e^{-.000436 \cdot t}$

½ of start

$\frac{1}{2} = e^{-.000436}$

$\ln(\frac{1}{2}) = -.000436$

$\frac{\ln(\frac{1}{2})}{-.000436} = \boxed{t = 1589.8}$

D) **The depreciation of the value for a car is modeled by the equation $y = 100{,}000(.85)^x$ for x years since 2000.**

i) **In what year was the value of the car was \$61,412.50?**

$\frac{100{,}000(.85)^x}{100{,}000} = \frac{61{,}412.50}{100{,}000}$

$.85^x = .614125$

$\log_{.85}(.614125) = x$

Change base

$\frac{\log(.614125)}{\log(.85)} = 3$

3years

$\boxed{2003}$

ii) **In what year, will the value of the car reach ¼ of its original value.**

$25{,}000 = 100{,}000(.85)^x$

$\frac{1}{4} = (.85)^x$

$\log_{.85}(\frac{1}{4}) = x \rightarrow \frac{\log(\frac{1}{4})}{\log(.85)} = \boxed{x = 8.5300}$

E) **A new automobile is purchased for \$20,000. If V = 20,000(0.8)^x, gives the car's value after *x* years, *about* how long will it take for the car to be worth \$8,200?**

$8{,}200 = 20{,}000(.8)^x$

$.41 = (.8)^x$

$\log_{.8}(.41) = x$

$\frac{\log(.41)}{\log(.8)} = \boxed{x = 3.995 \text{ years}}$

$(1-.1) = .9$

F) A cup of coffee contains 140 mg of caffeine. If caffeine leaves the body at 10% per hour, how long will it take for half of the caffeine to be eliminated from ones body?

$140e^{(-.1 \cdot t)} = 70$

$e^{-.1t} = \frac{1}{2} \rightarrow \ln(\frac{1}{2}) = -0.1t$

$\boxed{t = 6.9315 \text{ hours}}$

Identify initial value, growth or decay, growth/decay factor and growth/decay rate.

1) $4(2)^x$ — 4, growth, 2, 1 • 100%

2) $\frac{1}{2}(.75)^x$ — $\frac{1}{2}$, decay, .75, .25

Solve:

3) \$1200 is placed in an account at 4% compounded monthly for 2 years. It is then withdrawn at the end of the two years and placed in another bank at the rate of 5% compounded annually for 4 years. What is the balance in the second account after the 4 years?

$1200(1+\frac{.04}{12})^{12 \cdot 4} = 1407.84 \rightarrow 1407.84(1+\frac{.05}{1})^{1 \cdot 4}$

\$1711.24

4) \$1200 is placed in an account at 4% compounded continuously for 2 years. What is the balance in the account after the 17 years?

$1200e^{.04 \cdot 2}$ ≈ 1299.94

$1200e^{.04 \cdot 17}$ ≈ 2368.65

$$P\left(1+\frac{r}{n}\right)^{t \cdot n}$$

compound

Use the change of base formula to evaluate each logarithm. Round to four decimal places.

1. $\log_3 7 = \frac{\log 7}{\log 3}$

2. $\log_9 0.4 = \frac{\log .4}{\log 9}$

3. $\log_7 4 = \frac{\log 4}{\log 7}$

4. $\log_{20} 125 = \frac{\log 125}{\log 20}$

5. $\log_6 94 = \frac{\log (94}{\log(6)}$

6. $\log_{0.5} 4 = \frac{\log 4}{\log .5}$

7. $\log_{15} 1250$ $\frac{\log 1250}{\log 15}$

8. $\log_4 0.55 = \frac{\log .55}{\log 4}$

9. $\log_{1/3} 0.015 = \frac{\log .015}{\log (\frac{1}{3})}$

10. $\log_{17} 2 = \frac{\log (2)}{\log(17)}$

Use the properties of logarithms to expand each of the following.

1. $\log_2 5x = \log_2 5 + \log_2 X$

2. $\log_8 x^4 = 4 \log_8 X$

3. $\log_3 \frac{5}{x} = \log_3 5 - \log_3 X$

4. $\ln \sqrt{z} = \frac{1}{2} \ln (z)$

$\sqrt{z} = z^{\frac{1}{2}}$

5. $\ln z(z-1)^2 =$ $\ln z + 2\ln(z-1)$

6. $\log_7 \frac{x^2}{y^2 z^3} =$ $2\log_7 x - (2\log_7 y + 3\log_7 z)$

7. $\log\left(\frac{x^2-1}{x^3}\right)^3$ $3\left[\log(x^2-1) - 3\log(x)\right]$

8. $\log_x \frac{\sqrt{a}y^4}{z^4} =$ $(\frac{1}{2}\log_x a + 4\log_x y) - \log_x z$

9. $\ln \frac{x}{\sqrt{x^2+1}} =$ $\ln(x) - \frac{1}{2}(\ln(x^2+1)$

10. $\log(x^2 - 8x + 15) =$ Can't be expanded

Use the properties of logarithms to write the following as a single logarithm. Always check for like bases.

1. $\ln x + \ln 2 =$ $\ln(2x)$

2. $\log_4 z - \log_4 y =$ $\log_4\left(\frac{z}{y}\right)$

3. $2\log_2(x+4) =$ $\log_2(x+4)^2$

4. $\frac{1}{3}\log_3 5x =$ $\log_3\left((5x)^{1/3}\right)$

5. $\log_3(x-2)-\log_3(x+2) =$ $\log_3\left(\frac{(x-2)}{(x+2)}\right)$

6. $2\ln 8 + 5\ln z =$ $\ln(8^2 z^5) \Rightarrow \ln(64z^5)$

7. $3\ln 8 + 2\ln y - 4\ln z =$ $\ln\left(\frac{8^3 y^2}{z^4}\right) = \ln\left(\frac{512y^2}{z^4}\right)$

8. $4[\ln z + \ln(z+5)] - 2\ln(z-5) =$ $\ln\left(\left(\frac{z(z+5)}{(z-5)^2}\right)^4\right)$

9. $\ln x - 2[\ln(x+2)+\ln(x-2)] =$ $\ln\left(\frac{x}{(x+2)(x-2)}\right)$

10. $\frac{3}{2}\log_4 5t^6 - \frac{3}{4}\log_4 t^4 =$ $\log_4\left(\frac{(5t^6)^{3/2}}{(t^4)^{3/4}}\right) = \log_4\left(\frac{5^{3/2}t^9}{t^3}\right)$

$= \boxed{\log_4 \sqrt{5^3}\, t^6}$