**SOLUTIONS**

1. Conditional Probability Problem--Football (see spreadsheet <<PROBTAB1>>)

 a. P(R) = 0.70 b. P(L) = 0.30 c. P(B | R) = 0.20

 d. P(S | R) = 0.80 e. P(B | L) = 0.90 f. P(S | L) = 0.10

 Probability Table

 R L

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 S 0.56 0.03 | 0.59

 B 0.14 0.27 | 0.41

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 0.70 0.30 | 1.00

 Looking at the completed table does not give a sense of the logical order in which the numbers were entered. Here is the order and rationale:

 1.00 lower right corner total is always 1

 0.70 given

 0.30 given

 0.14 given that 20% of the R plays are B

 0.56 given that 80% of the R plays are S

 0.27 given that 90% of the L plays are B

 0.03 given that 10% of the L plays are S

 0.59 first-row total

 0.41 second-row total

 P (L | B) = 0.27 / 0.41 = 0.6585 P (R | B) = 0.14 / 0.41.= 0.3415

 Note that this is a pair of conditional complementary probabilities.

 P (L | S) = 0.03 / 0/59 = 0.0508 P (R | S) = 0.56 / 0/59 = 0.9492

 Another pair of conditional complementary probabilities.

 Are X's stance and play direction independent? Explain.

 No, they are dependent. R and S are positively related, as are B and L.

 P(R | S) > P(R) and P(B | L) > P(B)

 R and B are negatively related, as are S and L.

 P(R | B) < P(R) and P(S | L) < P(S)

 You coach the defense: "Always expect a right-side play." What percentage of the time will they be correct? 70% of the time (it is given that 70% of plays are R).

 You coach the defense: "If player X is balanced, expect a left-side play; if player X is shifted, expect a right-side play." What percentage of the time will they be correct?

 83% of the time. (Add the probabilities in the two cells that correspond to the positive-dependency combinations.)

2. Conditional Probability Problem--Medical Screening Test

 Create a probability table with columns D (disease) and W (well), and rows P (positive test) and N (negative test).

 Probability Table

 D W

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 P 0.00098 0.01998 | 0.02096

 N 0.00002 0.97902 | 0.97904

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 0.00100 0.99900 | 1.00000

 Looking at the completed table does not give a sense of the logical order in which the numbers were entered. Here is the order and rationale:

 1.00000 lower right corner total is always 1

 0.00100 given

 0.99900 the complement of 0.00100

 0.00098 given that 98% of the D people get P test results

 0.00002 2% of the D people must get N test results (false negatives)

 0.97902 given that 98% of the W people get N test results

 0.01998 2% of the W people must get P test results (false positives)

 0.02096 P row total

 0.97904 N row total

 Suppose you undergo the screening test and the result is positive. What is the probability that you actually have the disease? What if the test is negative?

 P(D | P) = 0.00098 / 0.02096 = 0.04676

 P(D | N) = 0.00002 / 0.97904 = 0.00002043

 So even if the test is positive, the probability of actually being sick is still quite low.

3. Conditional Probability Problem--Three Envelopes

 This one may be done with a probability table. Symbols:

 S1: first bill out of the envelope is a $1 bill

 S2: second bill out of the envelope is a $1 bill

 C1: first bill out of the envelope is a $100 bill

 C2: second bill out of the envelope is a $100 bill

 We are looking for P(C2 | C1)

 S1 C1

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 S2 0.333 0.167 | 0.500

 C2 0.167 0.333 | 0.500

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 0.500 0.500 | 1.000

 Looking at the completed table does not give a sense of the logical order in which the numbers were entered. Here is the order and rationale:

 1.000 lower right corner total is always 1

 0.500 when the first bill is taken out, there is a 50/50 chance between being a $100 bill and a $1 bill, since half of the bills are singles and half are hundreds

 0.500 the complement of 0.500

 0.333 P(S1 ∩ S2) is 1/3 since one of the three envelopes has two singles

 0.333 P(C1 ∩ C2) is 1/3 since one of the three envelopes has two hundreds

 0.167 by subtraction in the S1 column

 0.167 by subtraction in the C1 column

 0.500 S2 row total

 0.500 C2 row total

 What is the probability that the other bill is $100?

 P(C2 | C1) = 0.333 / 0.500 = 0.667

 Nearly everyone answers this question "50%," but they are wrong. Here's another way of looking at it. When the envelope is selected, there is a 2/3 chance, 0.667, that the two bills in it match. So when the first bill taken out is $100, there is a 0.667 chance that the other will match it and also be $100.

 In the three-card case, when the card is selected, there is a 2/3 chance that the card has matching sides. So whatever color is visible, there is a 2/3 chance that the other side is the same.

4. Conditional Probability Problem--Three Diseases (see spreadsheet <<PROBTAB2>>

 If a person has symptoms, what are the probabilities that he/she has disease A? B? C? If a person has no symptoms, what are the probabilities that he/she has disease A? B? C?

 A B C W

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 S 0.00900 0.00475 0.01500 0.00000 | 0.02875

 NS 0.00100 0.00025 0.00500 0.96500 | 0.97125

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 0.01000 0.00500 0.02000 0.96500 | 1.00000

 Looking at the completed table does not give a sense of the logical order in which the numbers were entered. Here is the order and rationale:

 1.00000 lower right corner total is always 1

 0.01000 given

 0.00500 given

 0.02000 given

 0.96500 by subtraction in the total row--missing number to equal 1

 0.00100 given that 10% of the A victims have no symptoms

 0.00900 by subtraction in the A column

 0.00025 given that 5% of the B victims have no symptoms

 0.00475 by subtraction in the B column

 0.00500 given that 25% of the C victims have no symptoms

 0.01500 by subtraction in the C column

 0.00000 given that symptoms are present only when one of the diseases is present--this would make P(S ∩ W) equal to zero

 0.96500 by subtraction in the W column

 0.02875 by addition in the S row

 0.97125 by addition in the NS row

P(A | S) = 0.00900 / 0.02875 = 0.3130 P(A | NS) = 0.00100 / 0.97125 = 0.001030

P(B | S) = 0.00475 / 0.02875 = 0.1652 P(B | NS) = 0.00025 / 0.97125 = 0.000257

P(C | S) = 0.00150 / 0.02875 = 0.5217 P(C | NS) = 0.00500 / 0.97125 = 0.005148

P(W | S) = 0.00000 / 0.02875 = 0.0000 P(W | NS) = 0.96500 / 0.97125 = 0.993600

Note that the numbers in each column add to 1. Each column is a conditional-probability partition. Consider the implication of the results for people with symptoms. If the diseases have different diagnostic tests, in what order should the diseases be tested for? Consider the implication of the results for people without symptoms. The probabilities of being sick seem reassuringly low for an individual, but what about for a city of a million people?